Accounting for persistence and volatility of good-level real exchange rates: the role of sticky information*

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Abstract

The volatile yet persistent real exchange rates can be observed not only in aggregate series but also in the individual good level data. Kehoe and Midrigan (2007) recently showed that, under a standard assumption on the nominal price stickiness, empirical frequencies of micro price adjustment cannot replicate the time-series properties of the international law-of-one-price deviations. We extend the Kehoe-Midrigan model to allow for the case when only a fraction of the firms update the information when their optimal reset prices are computed in the sense of Mankiw and Reis (2002). Under a reasonable assumption on the money growth process, we show that the model fully explains both persistence and volatility of the good-level real exchange rates. Furthermore, our model can allow presence of multiple cities in a country. Using a panel of U.S.-Canadian city pairs, we estimated a dynamic price adjustment process for each 165 individual goods. The empirical result suggests that the dispersion of average time of information update across goods is comparable to that of average time of price adjustment.

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1 Introduction

The behavior of aggregate real exchange rates has long attracted considerable attention, because their persistence and volatility are much higher than what economists believe is consistent with plausible degree of price rigidity. A sticky price model, a workhorse model in the field of the *New Open Economy Macroeconomics*, offers a convenient theoretical framework on the relationship between the price stickiness and real exchange rate behavior. Simulation results suggest that this model requires substantial price stickiness to match the persistence in the data (Chari, Kehoe and McGrattan (2002, CKM)). Unfortunately, however, micro studies on the frequencies of price changes for individual goods do not seem to support the explanation from a very slow price adjustment. In the U.S., a well-known study by Bils and Klenow (2004) shows the relatively fast price adjustment with the median monthly frequency of price changes of 26 percent and median duration between price changes of 4.3 months. A natural next step towards the final goal of explaining the aggregate real exchange rate anomaly is to relate it directly to the empirical literature on the micro price adjustment. An important contribution along this line is first made by a recent work of Kehoe and Midrigan (2007). Using simple testable implications derived from a standard Calvo-type sticky price model, they formally show that the observed empirical frequencies of micro price adjustment is too high to replicate the persistence and volatility of the real exchange rate at the individual good level.

In this paper, we examine this individual good version of the real exchange rate anomaly, and seek for a possible explanation for both persistence and volatility under the framework of sticky price models. In particular, we extend the Kehoe-Midrigan model to allow for the information stickiness, namely, the case when only a fraction of the firms updates the information each time they compute their optimal reset prices. In the macroeconomic literature, Mankiw and Reis (2002) showed that a model of information stickiness, or inattentiveness, is capable of explaining the observed slow response of aggregate inflation to monetary shocks much better than the pure sticky price model. When the information stickiness is added to the Calvo-type sticky price model framework, instead of replacing them, less frequent information update leads to higher price persistence, even if the price stickiness remains at the same level (Dupor, Kitamura and Tsuruga (2006, DKT)). With a plausible assumption on the money growth processes of two countries in the international setting, a similar effect also takes place to increase the persistence of real exchange rates. Using a panel of U.S.-Canadian city pairs, we show that our model can fully explain both persistence and volatility of the real exchange rates for each of 165 individual goods under consideration.

In addition to the generalization of the sticky price model to allow for the information stickiness, our analysis differs from Kehoe and Midrigan (2007) in several aspects. First, our empirical analysis is based on an international survey dataset which contains the information on retail price in local currency for highly disaggregated individual goods with fairly comprehensive coverage. As an advantage, more number of products (165) can be included in the analysis than 66 products used in Kehoe and Midrigan (2007). In addition, since the survey is conducted by a single agency, the Economist Intelligence Unit, we can expect a reasonable uniformity in the quality of the products among international cities. One limitation of our data is that it is sampled only annually with total of 16 time-series observation between 1990 and 2005. As in the case of Crucini and Shintani (2007), the difficulty of estimating persistence with short time-series, however, can be solved by utilizing the dynamic panel feature of the data.

Second, our theoretical model allows for the presence of multiple cities in a country. The model predicts that the size of long-run deviation of prices between the cross-border city pair can be different among pairs. For each good, we use the panel of 52 U.S.-Canadian city pairs to estimate a dynamic panel model and to compute the volatility under the error components model framework.

Third, we also examine the effect of the exclusion of sales on the performance of sticky price model in explaining the real exchange rate dynamics. Recently, Nakamura and Steinsson (2007) claim that the evidence of the fast price adjustment obtained by Bils and Klenow (2004) may be reflecting the presence of sales, or temporary price reduction. Nakamura and

Steinsson (2007) define the regular price change by excluding sales from the observed price change, and report that the median frequency of regular price changes increases to 8 to 11 months. Since stickier price based on their new definition of price change works in favor of sticky price explanation of real exchange rate, we evaluate the performance of the model using both of two alternative definitions of price change frequency.

The main finding of Kehoe and Midrigan (2007) turns out to be quite robust to the change in the data. We confirm that the both persistence and volatility are much higher than the prediction of a standard Calvo-type sticky price model even if we use (i) more disaggregated retail price data, (ii) the panel data which consists of multiple cities from two countries, and (iii) the frequency of price changes after the removal of sales.

Our extension of a standard Calvo-type sticky price model to include the information stickiness can fully account for both persistence and volatility, when the average duration between information updates is 14 to 17 months if sales are not removed, and 9 to 12 months if sales are removed. On the whole, the estimated values on the information delay are consistent with the previous results based on both aggregate and survey data. Furthermore, our empirical result suggests that the dispersion of average duration between information updates across goods can be comparable to that of average duration between price changes.

The ability of our model in fully replicating the observed persistence and volatility contrasts to another possible extension of the base-line sticky price model allowing for pricing complementarities. Kehoe and Midrigan (2007) also showed that such an extension only leads to a modest improvement in explaining the persistence and little improvement in explaining the variance.

This paper is organized as follows: Section 2 presents our models of good-level real exchange rates as a generalization of Kehoe and Midrigan's model. We then examine their implications to the time series properties of the good-level real exchange rates under alternative assumptions. Section 3 describes our dataset for the empirical analysis and explains how to exploit the data to evaluate our models. Section 4 presents results on the benchmark sticky price model and our extended model. The study ends with a section of what we believe is useful for future research.

2 The models of good-level real exchange rate

Consider an infinite horizon two country model with cash-in-advance constraint. The model consists of a home (e.g., the U.S.) country and a foreign (e.g., Canada) country. The home country has different local markets (e.g., Los Angeles, New York, Washington D.C., etc. in the U.S.). Analogously, the foreign country has different local markets (e.g., Montreal, Toronto, Vancouver, etc. in Canada). Each local market is monopolistically competitive and the goods are differentiated among locations. Firms that sell a good in a country set their price in the local currency to satisfy the demand for the good. Throughout this section, we assume that the unit of time is monthly.

The constant elasticity of substitution (CES) indexes aggregate a continuum of goods in three stages. First, the aggregate consumption in the home country c_t is a composite of each type of a particular good $j \in [0,1]$:

$$c_t = \left[\int c_t(j)^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}}. \tag{1}$$

Similarly, the aggregate consumption in the foreign country c_t^* is given by

$$c_t^* = \left[\int c_t^*(j)^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}}, \tag{2}$$

where $c_t(j)$ and $c_t^*(j)$ are the consumption of goods j in the home or the foreign country, respectively.

Second, the consumptions for good j, $c_t(j)$ and $c_t^*(j)$, are also a composite of goods j in different locations $l \in [0,1]$ and $l^* \in [0,1]$, respectively:

$$c_t(j) = \left[\int c_t(j,l)^{\frac{\theta-1}{\theta}} dl\right]^{\frac{\theta}{\theta-1}}, \tag{3}$$

and

$$c_t^*(j) = \left[\int c_t^*(j,l^*)^{\frac{\theta-1}{\theta}} dl^*\right]^{\frac{\theta}{\theta-1}}, \tag{4}$$

where $c_t(j,l)$ and $c_t^*(j,l^*)$ are consumption demanded for good j in location l in the home country and consumption demanded for good j in location l^* in the foreign country, respectively. For example, the index l may refer to Los Angeles, New York, or Washington D.C. The index l^* may refer to Montreal, Toronto, or Vancouver.

Finally, each type of goods j is traded internationally and aggregated by the CES index:

$$c_t(j,l) = \left[\int_0^1 c_t(j,l,z)^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}}, \tag{5}$$

and

$$c_t^*(j,l^*) = \left[\int_0^1 c_t^*(j,l^*,z)^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}}, \tag{6}$$

where $c_t(j,l,z)$ is a brand z that is categorized in good j and consumed in location l of the home country. Analogously, $c_t^*(j,l^*,z)$ is a brand z that is categorized in good j and consumed in location l^* of the foreign county. The index $z \in [0,1]$ denotes a brand for a particular good j. We assume that a brand $z \in [0,1/2]$ of a good j is produced in the home country and that a brand $z \in (1/2,1]$ of a good j is produced in the foreign country.

We assume complete markets for state-contingent money claims. This two country economy has one-period nominal bonds each of which corresponds to each event in period t. For simplicity, we assume that all of these bonds are denominated in the home currency.[1] Households in the home country hold B_{t+1} which depends on the state of the world in period $t+1$. Households in the foreign country hold B_{t+1}^* (denominated in the home currency) which also depends on the state of the world in period $t+1$. The price of such a bond is denoted by $Q_{t,t+1}$. Also, $Q_{t,t+h}$ would be the nominal stochastic discount factor by which both of the home and foreign firms discount their profits in period $t+h$ in period t.

2.1 Households

We follow Kehoe and Midrigan (2007) to model households' decision problem. Households in the home country maximize the discounted sum of $U(c_t,n_t) = \log c_t - \chi n_t$ ($\chi > 0$) subject to the resource and the cash-in-advance constraints. Their maximization problem is described

[1]As Kehoe and Midrigan (2007) argue, it does not matter if foreign consumers hold complete and state-contingent one-period nominal bonds denominated in the foreign currency. It would be simply a redundant assumption under state-contingent bond markets.

as follows:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, n_t), \tag{7}$$

$$\text{s.t.} \quad M_t + \mathbb{E}_t Q_{t,t+1} B_{t+1} = R_{t-1} W_{t-1} n_{t-1} + B_t + [M_{t-1} - P_{t-1} c_{t-1}] + T_t + \Pi_t, \tag{8}$$

$$M_t \geq P_t c_t. \tag{9}$$

Here, $0 < \beta < 1$ is the discount factor of the household and $\mathbb{E}_t(\cdot)$ denotes the expectation operator conditional on the information available in period t.

The left hand side of the period-by-period budget constraint (8) represents the nominal value of total wealth of the household brought in the beginning of the period $t+1$. It consists of cash holding M_t and the bond holdings B_{t+1}. As shown in the right hand of (8), the households receive nominal labor income $W_{t-1} n_{t-1}$ in period $t-1$ and earn its gross nominal interest rate R_{t-1} per unit of labor income until period t in terms of the home currency.[2] Households carry the nominal bond holding B_t and the remaining cash holding $(M_{t-1} - P_{t-1} c_{t-1})$ into period t, where P_t is the aggregate price index defined below. Finally, T_t and Π_t are nominal lump sum transfers from the government of the home country and nominal profits of firms in the home country, respectively.[3]

The equation (9) is the cash-in-advance constraint. The aggregate price P_t is given by $P_t = [\int P_t(j)^{1-\theta} dj]^{\frac{1}{1-\theta}}$, where $P_t(j)$ is the aggregate price index for good j over different locations: $P_t(j) = [\int P_t(j,l)^{1-\theta} dl]^{\frac{1}{1-\theta}}$. Finally, the price index for good j in a particular location l is given by

$$P_t(j,l) = \left[\int P_t(j,l,z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}}.$$

Households in the foreign country are analogously modeled except that they hold one-period nominal bonds denominated in the home currency. The period-by-period budget constraint of the foreign households is given by

$$M_t^* + \mathbb{E}_t Q_{t,t+1} \frac{B_{t+1}^*}{S_t} = \frac{S_{t-1} R_{t-1}}{S_t} W_{t-1}^* n_{t-1}^* + \frac{B_t^*}{S_t} + [M_{t-1}^* - P_{t-1}^* c_{t-1}^*] + T_t^* + \Pi_t^*,$$

[2]We assume that the government pays interest rate $R_t (= 1/\mathbb{E}_t Q_{t,t+1})$ on wage incomes in period t. This assumption allows households' intratemporal optimality condition to be undistorted.

[3]We assume that government's lump sum transfers and firms' profits in a country go to households in that country.

where S_t denotes the nominal exchange rate. As a result, the first order conditions of households in both countries are as follows:

$$\frac{W_t}{P_t} = \chi c_t \tag{10}$$
$$\frac{W_t^*}{P_t^*} = \chi c_t^* \tag{11}$$
$$\mathbb{E}_t Q_{t,t+1} = \beta \mathbb{E}_t \left(\frac{c_{t+1}}{c_t}\right)^{-1} \left(\frac{P_t}{P_{t+1}}\right) \tag{12}$$
$$\mathbb{E}_t Q_{t,t+1} = \beta \mathbb{E}_t \left(\frac{c_{t+1}^*}{c_t^*}\right)^{-1} \left(\frac{S_t P_t^*}{S_{t+1} P_{t+1}^*}\right) \tag{13}$$
$$M_t = P_t c_t \tag{14}$$
$$M_t^* = P_t^* c_t^*. \tag{15}$$

The equations (10) and (11) represent intratemporal substitution between labor and consumption while (12) and (13) represent intertemporal substitution between two different months. (12) and (13) are slightly different because foreign households buy state-contingent one-period nominal bonds denominated in the home currency. The equations (14) and (15) mean the cash-in-advance constraints always hold with equality.

We can derive several conditions from these first order conditions. First, the aggregate real exchange rate:

$$q_t = \frac{S_t P_t^*}{P_t} = \kappa \frac{c_t}{c_t^*}, \tag{16}$$

where $\kappa = q_0 c_0^*/c_0$.[4] Second, under the CIA constraints (14) and (15), (16) implies that the nominal exchange rate can be written as

$$S_t = \kappa \frac{M_t}{M_t^*}. \tag{17}$$

Finally, intratemporal conditions (10) and (11) reduce to

$$W_t = \chi M_t, \tag{18}$$
$$W_t^* = \chi M_t^*. \tag{19}$$

Thus, the nominal wage rate in a country is proportional to the household's money holdings in that country.

[4]From (12) and (13), we obtain $\frac{P_{t+1}^* c_{t+1}^*}{P_{t+1} c_{t+1}} S_{t+1} = \frac{P_t^* c_t^*}{P_t c_t} S_t$ in each event in period $t+1$. Because q_t is defined as $\frac{S_t P_t^*}{P_t}$, it immediately follows that $q_{t+1} \frac{c_{t+1}^*}{c_{t+1}} = q_t \frac{c_t^*}{c_t} = q_{t-1} \frac{c_{t-1}^*}{c_{t-1}} = \cdots = q_0 \frac{c_0^*}{c_0} = \kappa$.

2.2 Firms

Consider firms that produce output $y_t(j,z)$. They have the production function of the following form:

$$y_t(j,z) = n_t(j,z), \tag{20}$$

where $n_t(j,z)$ is labor input for firms that produce a brand z of a good j in the home country.

A home firm sells its goods in the home and foreign local markets. The firm's output must satisfy the following constraint:

$$\int_l c_t(j,l,z)dl + \int_{l^*} (1+\tau(j,l^*))c_t^*(j,l^*,z)dl^* = y_t(j,z), \tag{21}$$

where $\tau(j,l^*)$ is a transportation cost in exporting a good j from the home country to a location l^* of the foreign country. The transportation cost may reflect the border effect implied by tariff barrier and non tariff bureaucratic barrier imposed on foreign business. We assume that it depends on a good j and a location l^*.[5] This $\tau(j,l^*)$ means that firms require $(1+\tau(j,l^*))$ unit of a good j to export one unit of that good to a location l^* of the foreign country.

Foreign firms that produce output $y_t^*(j,z)$ have the same linear technology as home firms and transportation costs to export good j to local markets of the home country. The firm's production resource must satisfy

$$\int_l (1+\tau(j,l))c_t(j,l,z)dl + \int_{l^*} c_t^*(j,l^*,z)dl^* = y_t^*(j,z). \tag{22}$$

In what follows, we will consider nominal rigidities á la Calvo (1983) as a main source of slow adjustment in good-level real exchange rates as in Kehoe and Midrigan (2007). In addition to Kehoe and Midrigan's benchmark model, we will discuss two extensions in the following order. First, we follow CKM and consider the case where money growth follows an stochastic process slightly more general than the i.i.d. process. Second, more importantly, we further generalize the Calvo model by introducing the information stickiness into the model. We will discuss these extensions in turn.

[5]The assumption of the dependence is consistent with heterogeneous long-run deviation of good-level real exchange rates between cities.

2.3 The Calvo model

We model the nominal price rigidities as in Calvo (1983) and Yun (1996): only a fraction of firms $1-\lambda_j$ are allowed to reset the price every month.[6] Following Kehoe and Midrigan (2007), we allow the infrequency of price changes to vary according the type of good j but we assume that the infrequency of price changes is the same between the two countries.

All home firms that sell their good j in location l choose the same optimal price when they adjust prices in period t. The price $P_{H,t}(j,l)$ solves the following maximization problem:

$$\begin{aligned}
\max_{P_{H,t}(j,l)} \mathbb{E}_t &\sum_{h=0}^{\infty} \lambda_j^h Q_{t,t+h}[P_{H,t}(j,l) - W_{t+h}] \\
&\times \left(\frac{P_{H,t}(j,l)}{P_{t+h}(j,l)}\right)^{-\theta} \left(\frac{P_{t+h}(j,l)}{P_{t+h}(j)}\right)^{-\theta} \left(\frac{P_{t+h}(j)}{P_{t+h}}\right)^{-\theta} c_{t+h},
\end{aligned} \tag{23}$$

for all location $l \in [0,1]$. Here, we used the three demand functions as constraints:

$$\begin{aligned}
c_t(j) &= \left(\frac{P_t(j)}{P_t}\right)^{-\theta} c_t \\
c_t(j,l) &= \left(\frac{P_t(j,l)}{P_t(j)}\right)^{-\theta} c_t(j) \\
c_t(j,l,z) &= \left(\frac{P_t(j,l,z)}{P_t(j,l)}\right)^{-\theta} c_t(j,l).
\end{aligned}$$

The optimality condition for $P_{H,t}(j,l)$ is

$$\begin{aligned}
&\mathbb{E}_t \sum_{h=0}^{\infty} \lambda_j^h Q_{t,t+h} \left(\frac{P_{H,t}(j,l)}{P_{t+h}}\right)^{-\theta} c_{t+h} \\
=&\frac{\theta}{\theta-1}\mathbb{E}_t \sum_{h=0}^{\infty} \lambda_j^h Q_{t,t+h} \left(\frac{W_{t+h}}{P_{H,t}(j,l)}\right) \left(\frac{P_{H,t}(j,l)}{P_{t+h}}\right)^{-\theta} c_{t+h}.
\end{aligned} \tag{24}$$

Similarly, all foreign firms that export and sell their good j in location l choose the same optimal price $P_{F,t}(j,l)$ when they adjust prices. The price $P_{F,t}(j,l)$ for these firms solves the maximization problem:

$$\begin{aligned}
\max_{P_{F,t}(j,l)} \mathbb{E}_t &\sum_{h=0}^{\infty} \lambda_j^h Q_{t,t+h}[P_{F,t}(j,l) - (1+\tau(j,l))S_{t+h}W^*_{t+h}] \\
&\times \left(\frac{P_{F,t}(j,l)}{P_{t+h}(j,l)}\right)^{-\theta} \left(\frac{P_{t+h}(j,l)}{P_{t+h}(j)}\right)^{-\theta} \left(\frac{P_{t+h}(j)}{P_{t+h}}\right)^{-\theta} c_{t+h},
\end{aligned} \tag{25}$$

[6] As we will discuss in the empirical section, the data on empirical frequency of micro price adjustment is available only in monthly frequency.

for all location $l \in [0,1]$.

The optimality condition is of the form similar to (24):

$$
\begin{aligned}
&\mathbb{E}_t \sum_{h=0}^{\infty} \lambda_j^h Q_{t,t+h} \left(\frac{P_{F,t}(j,l)}{P_{t+h}} \right)^{-\theta} c_{t+h} \\
=&\frac{\theta}{\theta-1} \mathbb{E}_t \sum_{h=0}^{\infty} \lambda_j^h Q_{t,t+h} \left(\frac{(1+\tau(j,l))S_{t+h}W^*_{t+h}}{P_{F,t}(j,l)} \right) \left(\frac{P_{F,t}(j,l)}{P_{t+h}} \right)^{-\theta} c_{t+h}.
\end{aligned} \tag{26}
$$

2.3.1 Equilibrium of the Calvo model

We assume that the monetary authorities set the growth rates of the money stock following an AR(1) process of the form:

$$
\log \mu_t = \rho \log \mu_{t-1} + \varepsilon_t, \tag{27}
$$

$$
\log \mu_t^* = \rho \log \mu_{t-1}^* + \varepsilon_t^*, \tag{28}
$$

where ε_t and ε_t^* are mean-zero i.i.d shock and $\mu_t = M_t/M_{t-1}$ and $\mu_t^* = M_t^*/M_{t-1}^*$ are the growth rate of the money supply in the two countries. For simplicity, we assume that the steady state of the money growth rates is one and that the persistence parameter $\rho \geq 0$ is common in both countries.

We assume that total transfer from the government in the home country equals home money injection minus the lump sum tax from the government paying interest: $T_t = M_t - M_{t-1} - (R_{t-1} - 1)W_{t-1}n_{t-1}$. Similarly, the total transfer in the foreign country is of the similar form: $T_t^* = M_t^* - M_{t-1}^* - (\frac{S_{t-1}R_{t-1}}{S_t} - 1)W_t^* n_t^*$.

The total profits of home firms are exclusively given to households in the home country. In other words, $\Pi_t = \int_j \int_{z=0}^{\frac{1}{2}} \Pi_t(j,z)dzdj$, where $\Pi_t(j,z)$ is the profit of a home firm. Similarly, the total profits of foreign firms are given to households in the foreign county: $\Pi_t^* = \int_j \int_{z=\frac{1}{2}}^{1} \Pi_t^*(j,z)dzdj$, where $\Pi_t^*(j,z)$ is the profit of a foreign firm.

Next, the market clearing condition for good markets has been given by (21) and (22). As for the labor markets in the two countries, we have

$$
\begin{aligned}
n_t &= \int_j \int_{z=0}^{\frac{1}{2}} n_t(j,z)dzdj, \\
n_t^* &= \int_j \int_{z=\frac{1}{2}}^{1} n_t^*(j,z)dzdj.
\end{aligned}
$$

Finally, the bond market clearing condition is $B_t + B_t^* = 0$ for all t.

An *equilibrium* of this economy is a collection of allocations and prices:

- $\{c_t(j,l,z)\}_{j,l,z}$, M_t, B_{t+1}, n_t for households in the home country;
- $\{c_t^*(j,l^*,z)\}_{j,l,z}$, M_t^*, B_{t+1}^*, n_t^* for households in the foreign country;
- $\{P_t(j,l,z), P_t^*(j,l^*,z), n_t(j,z), y_t(j,z)\}_{j,l,z\in[0,1/2]}$ for firms in the home country;
- $\{P_t(j,l,z), P_t^*(j,l^*,z), n_t^*(j,z), y_t^*(j,z)\}_{j,l^*,z\in(1/2,1]}$ for firms in the foreign country;
- Nominal wages and bond prices satisfy the following conditions:
    1. Households' allocations solve their maximization problem;
    2. Prices and allocations of firms solve their maximization problem (23) and (25);
    3. All markets clear;
    4. The money supply process and transfers satisfy the specification above.

2.3.2 Implications on the good-level real exchange rates

We now discuss implications of the Calvo model under slightly more generalized setting than Kehoe and Midrigan (2007). In contrast to Kehoe and Midrigan (2007) assuming an i.i.d money growth ($\rho = 0$), our model allows the case of an AR(1) money growth ($\rho > 0$).

Log-linearization (24) around the steady state yields the (log) optimal price for home firms that reset prices in period t:

$$\hat{P}_{H,t}(j,l) = (1-\lambda_j\beta)\sum_{h=0}^{\infty}(\lambda_j\beta)^h \mathbb{E}_t \hat{M}_{t+h}, \tag{29}$$

where $\hat{P}_{H,t}(j,l)$ and $\hat{M}_t$ are the log-deviation of $P_{H,t}(j,l)$ and M_t from the steady state, respectively. Here, we use the proportionality of nominal wages to money supply (i.e., (18)) to replace the log-deviation of W_t with $\hat{M}_t$ (i.e., $\hat{W}_t = \hat{M}_t$). Thus, the firms that adjust prices in period t choose their price so as to equalize it to the weighted average of the current and future path of nominal marginal costs.

Analogously, we can derive the log-deviation of optimal price for foreign firms from (26):

$$\hat{P}_{F,t}(j,l) = (1-\lambda_j\beta)\sum_{h=0}^{\infty}(\lambda_j\beta)^h\mathbb{E}_t[\hat{S}_{t+h} + \hat{M}^*_{t+h}].$$

However, from (17),

$$\hat{P}_{F,t}(j,l) = (1-\lambda_j\beta)\sum_{h=0}^{\infty}(\lambda_j\beta)^h\mathbb{E}_t[\hat{M}_{t+h}]. \tag{30}$$

Thus, $\hat{P}_{F,t}(j,l)$ equals $\hat{P}_{H,t}(j,l)$ under our specific preference assumption.

Given $\hat{P}_{F,t}(j,l) = \hat{P}_{H,t}(j,l)$, the log-deviation of price index for $\hat{P}_t(j,l)$ under the Calvo model can be written as

$$\hat{P}_t(j,l) = \lambda_j\hat{P}_{t-1}(j,l) + (1-\lambda_j)\hat{P}_{H,t}(j,l).$$

It is convenient to normalize $\hat{P}_{H,t}(j,l)$ (and $\hat{P}_t(j,l)$) by $\hat{M}_t$ to assure stationarity. In particular, let $\hat{p}_{H,t}(j,l) = \hat{P}_{H,t}(j,l) - \hat{M}_t$ and $\hat{\mu}_t = \hat{M}_t - \hat{M}_{t-1}$. Then,

$$\begin{aligned}
\hat{p}_{H,t}(j,l) &= (1-\lambda_j\beta)\sum_{h=0}^{\infty}(\lambda_j\beta)^h\mathbb{E}_t[\hat{M}_{t+h} - \hat{M}_t] \\
&= (1-\lambda_j\beta)\sum_{h=0}^{\infty}(\lambda_j\beta)^h\mathbb{E}_t\left[\sum_{r=1}^{h}\hat{\mu}_{t+r}\right] \\
&= (1-\lambda_j\beta)\sum_{h=0}^{\infty}(\lambda_j\beta)^h\rho\left[\frac{1-\rho^h}{1-\rho}\right]\hat{\mu}_t \\
&= \left[\frac{\lambda_j\beta\rho}{1-\lambda_j\beta\rho}\right]\hat{\mu}_t.
\end{aligned} \tag{31}$$

Note that $\hat{p}_{F,t}(j,l) = \hat{P}_{F,t}(j,l) - \hat{M}_t = \hat{p}_{H,t}(j,l)$. Thus, the short-run dynamics of the optimal prices are the same between the prices set by firms in spite of the transportation costs and the country where productions occur. Next, using (31), we stationarize $\hat{P}_t(j,l)$ by $\hat{M}_t$ to get

$$\hat{p}_t(j,l) = \lambda_j\hat{p}_{t-1}(j,l) - \lambda_j\hat{\mu}_t + (1-\lambda_j)\left[\frac{\lambda_j\beta\rho}{1-\lambda_j\beta\rho}\right]\hat{\mu}_t, \tag{32}$$

where $\hat{p}_t(j,l) = \hat{P}_t(j,l) - \hat{M}_t$.

We use similar arguments to obtain the foreign price index for a good j and a location l^*:

$$\hat{p}^*_t(j,l^*) = \lambda_j\hat{p}^*_{t-1}(j,l^*) - \lambda_j\hat{\mu}^*_t + (1-\lambda_j)\left[\frac{\lambda_j\beta\rho}{1-\lambda_j\beta\rho}\right]\hat{\mu}^*_t. \tag{33}$$

Now, we define the real exchange rate for good j between locations l and l^* as $\hat{q}_t(j,l,l^*) = \log q_t(j,l,l^*) - \log q(j,l,l^*)$, where $q_t(j,l,l^*)$ is given by

$$q_t(j,l,l^*) = \frac{S_t P_t^*(j,l^*)}{P_t(j,l)}, \tag{34}$$

and $q(j,l,l^*)$ is its steady state value.

The next proposition characterizes the short-run good-level real exchange rate dynamics under the Calvo model with a slight generalization of Kehoe and Midrigan (2007).

Proposition 1. *(A generalization of Proposition 1 by Kehoe and Midrigan (2007)) Under the preference assumption $U(c,n) = \log c - \chi n$ and the assumption of money growth* (27) *and* (28)*, the stochastic process governing the good-level real exchange rate between any locations l and l^*, $\hat{q}_t(j,l,l^*)$ is of the form:*

$$\hat{q}_t(j,l,l^*) = (\lambda_j + \rho)\hat{q}_{t-1}(j,l,l^*) - \lambda_j \rho \hat{q}_{t-2}(j,l,l^*) + \theta_j \eta_t, \tag{35}$$

where $\hat{q}_t(j,l,l^*) = \hat{S}_t + \hat{P}_t^*(j,l^*) - \hat{P}_t(j,l)$, $\theta_j = \lambda_j - (1-\lambda_j)\frac{\lambda_j \beta \rho}{1-\lambda_j \beta \rho}$, and $\eta_t (= \varepsilon_t - \varepsilon_t^*)$ is i.i.d. with its variance σ_η^2. In other words, the good-level real exchange rate follows an AR(2) process.

Proof. From (16) and (17), $\hat{q}_t(j,l,l^*) = \hat{p}_t^*(j,l^*) - \hat{p}_t(j,l)$. Subtracting (32) from (33) yields $\hat{q}_t(j,l,l^*) = \lambda_j \hat{q}_{t-1} + \theta_j(\hat{\mu}_t - \hat{\mu}_t^*)$. Because $\hat{\mu}_t - \hat{\mu}_t^*$ follow an AR(1) from (27) and (28), we obtain (35) and proved Proposition 1. □

This proposition is a generalization of proposition 1 by Kehoe and Midrigan (2007). To see this, suppose that the money growth rates follow an i.i.d process ($\rho = 0$). Then, the equation (35) reduces to an AR(1) model with its coefficient λ_j and $\theta_j = \lambda_j$. This is exactly the same as their proposition 1.

In what follows, we use the sum of autoregressive coefficients (SAR) to measure the persistence of real exchange rates. Theoretically, the SAR has one-to-one relationship to the cumulative long-run impulse response to a shock in time series. Practically, it is computationally simple and has been often used as a persistence measure in applications (e.g., Andrews and Chen (1994) and Clark (2006)). In our case, the persistence measured by the SAR corresponds to $\lambda_j + \rho - \lambda_j \rho$ when $\rho > 0$ (AR(2)) and becomes λ_j when $\rho = 0$ (AR(1)).

The left panel of Fig.1 shows the effect of increasing ρ on the persistence for the two representative goods - a good with relatively slow price adjustment ($\lambda_j = 0.95$) and a good with relatively fast price adjustment ($\lambda_j = 0.5$). The SAR is strictly increasing in ρ regardless of the degree of price stickiness under $\lambda_j \in [0, 1)$.

The intuition behind the persistent dynamics is straightforward. Note that the relationship between the good-level exchange rate and the nominal exchange rate growth is given by

$$\hat{q}_t(j, l, l^*) = \lambda_j \hat{q}_{t-1}(j, l, l^*) + \theta_j \Delta \hat{S}_t, \tag{36}$$

where $\Delta \hat{S}_t = \hat{\mu}_t - \hat{\mu}_t^*$ from (17). If $\rho = 0$ as in Kehoe and Midrigan (2007), $\Delta \hat{S}_t$ is an i.i.d shock. Thus, the good-level real exchange rate follows AR(1) and does not have any additional source to increase the persistence. In contrast, if $\rho > 0$ as in CKM, the persistence of monetary shocks to $\Delta \hat{S}_t$ increases the persistence in the real exchange rates.

Next, we turn to the issue of the predicted real exchange rate volatility. To assess the prediction of the model, it is convenient to consider the effect of change in the standard deviation of ΔS_t on that of $q_t(j, l, l^*)$. For the simplest benchmark case with $\rho = 0$, the standard deviation of $\hat{q}_t(j, l, l^*)$ can be written by $f_1(\lambda_j)\text{std}(\Delta S_t)$, where $f_1(\lambda_j) = \lambda_j / \sqrt{1 - \lambda_j^2}$. If we call $f_1(\lambda_j)$ as the scaling factor for volatility, a larger scaling factor corresponds to the higher predicted real exchange rate volatility for the same volatility level of the nominal exchange rate growth. Such a scaling factor is useful in evaluating the effect of change in parameter values on the predicted volatility. In this simplest case, because $f_1(\lambda_j)$ is strictly increasing in λ_j, the real exchange rate of a good with larger λ_j should be more volatile. As for the AR(2) case, we can derive the standard deviation of a real exchange rate of the form:

$$\begin{aligned}
\text{std}(\hat{q}_t(j, l, l^*)) &= \tilde{f}_2(\lambda_j, \rho, \beta)\text{std}(\eta_t) \\
&= \tilde{f}_2(\lambda_j, \rho, \beta)\sqrt{1 - \rho^2}\text{std}(\Delta S_t) \\
&= f_2(\lambda_j, \rho, \beta)\text{std}(\Delta S_t),
\end{aligned}$$

where $\tilde{f}_2(\lambda_j, \rho, \beta)$ can be easily obtained from an AR(2) process using the variance formula of AR(2) process and the third equality follows from $\text{std}(\Delta S_t) = \text{std}(\eta_t)/\sqrt{1 - \rho^2}$. Here, the scaling factor for the AR(2) is $f_2(\lambda_j, \rho, \beta)$.

Now, how does a positive ρ affect the volatility of good-level real exchange rates? Unfortunately, the effect on the volatility is ambiguous. The right panel of Fig.1 plots the scaling factors $f_2(\lambda_j, \rho, \beta)$ based on different λ_j.[7] The figure suggests that the volatility is increasing in ρ when the price change is relatively infrequent (e.g., $\lambda_j = 0.95$) but as ρ increases, the volatility turns to be decreasing in ρ.[8] On the other hand, the volatility measure is actually decreasing in ρ, when the price change is relatively frequent. (e.g., $\lambda_j = 0.5$.) Thus, we have the ambiguous effect of a positive ρ on the volatility of the good-level real exchange rate.

Fig.2 plots the SAR and the scaling factors over λ_j. The figure compares two values of ρ. Kehoe and Midrigan (2007) calibrate $\rho = 0$. On the other hand, CKM calibrate a positive ρ by estimating from the U.S. data for M1. Following CKM, we use $\rho = 0.83$ in monthly basis.[9]

As shown in the left panel of Fig.2, the effect of a positive ρ on the SAR is clear. When $\rho = 0$ as in Kehoe and Midrigan (2007), the model predicts that the SAR should accord with λ_j, which means that the model predicts 45 degree line in this panel. On the other hand, when $\rho > 0$, the model predicts a flatter line. Thus, a high persistence of the money growth rates increases the persistence.

Once again, however, the effect of a positive ρ on the volatility is ambiguous. The model predicts that the scaling factor is lower for $\rho = 0.83$ than for $\rho = 0$ when price adjustment is fast. When the price adjustment is slow, we have a larger scaling factor for $\rho = 0.83$ than for $\rho = 0$.

We summarize our theoretical findings from the Calvo model as follows. We find that the serially correlated money growth as in CKM will generate the persistent good-level real exchange rate. However, its implications in terms of the volatility is rather ambiguous. In the next subsection, we introduce information stickiness into the Calvo model and show that the extended model can explain the volatility as well as persistence under the reasonable range of parameters.

[7]We set the discount factor β to 0.99.

[8]When ρ is high, $\sqrt{1-\rho^2}$ proceeding $\tilde{f}_2(\lambda_j, \rho)$ directly reduces the volatility.

[9]Their estimate of ρ is 0.68 in quarterly basis. We transform this quarterly persistence of M1 growth into the monthly persistence by solving $Cov(\hat{M}_t - \hat{M}_{t-3}, \hat{M}_{t-3} - \hat{M}_{t-6})/Var(\hat{M}_t - \hat{M_{t-3}}) = 0.68$ for ρ. We obtained the resulting monthly persistence of M1 money growth of 0.83.

2.4 Adding information stickiness: dual stickiness approach

We now add information stickiness á la Mankiw and Reis (2002) into the Calvo model. Consider firms facing two nominal rigidities. First, each firm has a constant probability of price resetting $1-\lambda_j$ like the Calvo model. Second, each firm also infrequently updates its information set with a constant probability of $1-\omega_j$ every month. Otherwise, firms have to use the old information set that it has last updated to determine prices. For simplicity, we assume that the two probabilities are independent each other.[10]

DKT develop this combined stickiness structure to explain persistent inflation dynamics as we specified above. In the DKT model, infrequent price changes arise due to the Calvo assumption of price changes. However, when firms compute their optimal reset prices, a fraction of firms use the newest information set and the remaining firms use the stale information set to determine prices. They call the model with two combined stickiness structure the "dual stickiness" model.

Following DKT, we introduce the information stickiness into the Calvo model and call the extended model the dual stickiness model. All home firms that sell their good j in location l choose different prices according to the information set they last updated. When firms adjust prices with the same information set, they set the same price. Let $P^k_{H,t}(j,l)$ be the optimal reset price conditional on the information set k month ago. The price $P^k_{H,t}(j,l)$ for the home firms solves

$$\begin{aligned}\max_{P^k_{H,t}(j,l)} \mathbb{E}_{t-k} \sum_{h=0}^{\infty} \lambda_j^h Q_{t,t+h}[P^k_{H,t}(j,l) - W_{t+h}] \\ \times \left(\frac{P^k_{H,t}(j,l)}{P_{t+h}(j,l)}\right)^{-\theta} \left(\frac{P_{t+h}(j,l)}{P_{t+h}(j)}\right)^{-\theta} \left(\frac{P_{t+h}(j)}{P_{t+h}}\right)^{-\theta} c_{t+h},\end{aligned} \tag{37}$$

for $k = 0, 1, 2, \cdots$ and for all locations $l \in [0,1]$.

[10]As emphasized in DKT, this information stickiness structure combined with price stickiness has an advantage over the case of sticky information alone. The combined model not only explains sluggish dynamics of the price index through information stickiness, but also exploits the stylized fact that many firms adjust prices infrequently. In fact, the original Mankiw-Reis model predicts that all firms should change prices by inflation when they last expected that it grows. In this case, there should not be essentially any infrequency of price changes.

The optimality condition for $P^k_{H,t}(j,l)$ is

$$
\begin{aligned}
&\mathbb{E}_{t-k}\sum_{h=0}^{\infty}\lambda_j^h Q_{t,t+h}\left(\frac{P^k_{H,t}(j,l)}{P_{t+h}}\right)^{-\theta} c_{t+h} \\
=&\frac{\theta}{\theta-1}\mathbb{E}_{t-k}\sum_{h=0}^{\infty}\lambda_j^h Q_{t,t+h}\left(\frac{W_{t+h}}{P^k_{H,t}(j,l)}\right)\left(\frac{P^k_{H,t}(j,l)}{P_{t+h}}\right)^{-\theta} c_{t+h},
\end{aligned}
\tag{38}
$$

for $k = 0, 1, 2, \cdots$. Foreign firms that sell their good j by exporting to a location l also choose prices based on their information set that they last updated. They choose prices so as to solve the maximization problem:

$$
\begin{aligned}
&\max_{P^k_{F,t}(j,l)} \mathbb{E}_{t-k}\sum_{h=0}^{\infty}\lambda_j^h Q_{t,t+h}[P^k_{F,t}(j,l)-(1+\tau(j,l))S_{t+h}W^*_{t+h}] \\
&\times\left(\frac{P^k_{F,t}(j,l)}{P_{t+h}(j,l)}\right)^{-\theta}\left(\frac{P_{t+h}(j,l)}{P_{t+h}(j)}\right)^{-\theta}\left(\frac{P_{t+h}(j)}{P_{t+h}}\right)^{-\theta} c_{t+h},
\end{aligned}
\tag{39}
$$

for $k = 0, 1, 2, \cdots$. The optimality condition is similar to (38):

$$
\begin{aligned}
&\mathbb{E}_{t-k}\sum_{h=0}^{\infty}\lambda_j^h Q_{t,t+h}\left(\frac{P^k_{F,t}(j,l)}{P_{t+h}}\right)^{-\theta} c_{t+h} \\
=&\frac{\theta}{\theta-1}\mathbb{E}_{t-k}\sum_{h=0}^{\infty}\lambda_j^h Q_{t,t+h}\left(\frac{(1+\tau(j,l))S_{t+h}W^*_{t+h}}{P^k_{F,t}(j,l)}\right)\left(\frac{P^k_{F,t}(j,l)}{P_{t+h}}\right)^{-\theta} c_{t+h},
\end{aligned}
\tag{40}
$$

for $k = 0, 1, 2, \cdots$.

2.4.1 Equilibrium of the dual stickiness model

An equilibrium of the dual stickiness model economy is not much different from the definition of the equilibrium of the Calvo model. Prices and allocations of firms solve the maximization problems (37) and (39) instead of (23) and (25).

2.4.2 Implications on the good level real exchange rates

We now discuss the implications of the dual stickiness model. Let $\hat{P}^k_{H,t}(j,l)$ be the log deviation of $P^k_{H,t}(j,l)$ from the steady state. Log-linearizing (38) around the steady state yields

$$
\hat{P}^k_{H,t}(j,l) = (1-\lambda_j\beta)\sum_{h=0}^{\infty}(\lambda_j\beta)^h\mathbb{E}_{t-k}\hat{M}_{t+k},
$$

for $k = 0, 1, 2, \cdots$.. This equation is similar to (29). Furthermore, the law of iterated expectations implies

$$\hat{P}^k_{H,t}(j,l) = \mathbb{E}_{t-k}\hat{P}_{H,t}(j,l).$$

Here, we used $\hat{P}^0_{H,t}(j,l) = \hat{P}_{H,t}(j,)$ because of the equivalence between (24) and (38) under $k = 0$.

Consider the weighted average of newly set prices that home firms choose when they adjust prices in period t. First, home firms choose $\mathbb{E}_{t-k}\hat{P}_{H,t}(j,l)$ according to their information they last updated. Second, foreign firms choose $\mathbb{E}_{t-k}\hat{P}_{F,t}(j,l)$. Once again, we can use the fact that $\hat{P}_{F,t}(j,l) = \hat{P}_{H,t}(j,l)$ under our preference assumption. Therefore, $\hat{P}^k_{F,t}(j,l) = \hat{P}^k_{H,t}(j,l)$ for $k > 0$, due to the law of iterated expectations. Finally, let $\hat{X}_t(j,l)$ be the weighted average for the newly set prices for good j in location l of the home country. This collects reset prices based on different information sets. Consequently, the weighted average of the newly set prices in period t for good j in location l is given by

$$\hat{X}_t(j,l) = (1-\omega_j)\sum_{k=0}^{\infty}\omega_j^k\mathbb{E}_{t-k}\hat{P}_{H,t}(j,l), \tag{41}$$

which simply takes the weighted average of the optimal reset price conditional on different information sets and is similar to the formulation of the price index in Mankiw and Reis (2002, p.1300).

Now, (41) can be rewritten as follows. By definition, $\hat{P}_{H,t}(j,l) = \Delta\hat{P}_{H,t}(j,l) + \hat{P}_{H,t-1}(j,l)$. Thus,

$$\begin{aligned}\hat{X}_t(j,l) = (1-\omega_j)\hat{P}_{H,t}(j,l) + \omega_j(1-\omega_j)\sum_{k=0}^{\infty}\omega_j^k\mathbb{E}_{t-k-1}\Delta\hat{P}_{H,t}(j,l) \\ + \omega_j(1-\omega_j)\sum_{k=0}^{\infty}\omega_j^k\mathbb{E}_{t-k-1}\hat{P}_{H,t-1}(j,l).\end{aligned}$$

The second line of the equation is $\omega_j\hat{X}_{t-1}(j,l)$ from (41). Hence,

$$\hat{X}_t(j,l) = \omega_j\hat{X}_{t-1}(j,l) + (1-\omega_j)\hat{P}_{H,t}(j,l) + \omega_j(1-\omega_j)\sum_{k=0}^{\infty}\omega_j^k\mathbb{E}_{t-k-1}\Delta\hat{P}_{H,t}(j,l).$$

To stationarize the variables in the equation, define $\hat{x}_t(j,l) = \hat{X}_t(j,l) - \hat{M}_t$. Then,

$$\begin{aligned}\hat{x}_t(j,l) = \omega_j\hat{x}_{t-1}(j,l) - \omega_j\hat{\mu}_t + (1-\omega_j)\hat{p}_{H,t}(j,l) \\ +\omega_j(1-\omega_j)\sum_{k=0}^{\infty}\omega_j^k\mathbb{E}_{t-k-1}[\Delta\hat{p}_{H,t}(j,l) + \hat{\mu}_t].\end{aligned} \tag{42}$$

Appendix A shows that we can derive the closed form solution to $\hat{x}_t(j,l)$:

$$\hat{x}_t(j,l) = \omega_j \hat{x}_{t-1}(j,l) + a_j \hat{\mu}_t + \frac{b_j}{1-\omega_j \rho L} \hat{\mu}_{t-1}, \tag{43}$$

where $a_j = \frac{\lambda_j \beta \rho - \omega_j}{1-\lambda_j \beta \rho}$, $b_j = \frac{\omega_j \rho (1-\lambda_j \beta)(1-\omega_j)}{1-\lambda_j \beta \rho}$, and L is the lag operator.

Next, we consider the price index for good j in location l under the dual stickiness model. The index can be written as

$$\hat{P}_t(j,l) = \lambda_j \hat{P}_{t-1}(j,l) + (1-\lambda_j) \hat{X}_t(j,l).$$

By normalization by $\hat{M}_t$, we get

$$\hat{p}_t(j,l) = \lambda_j \hat{p}_{t-1}(j,l) - \lambda_j \hat{\mu}_t + (1-\lambda_j) \hat{x}_t(j,l). \tag{44}$$

By similar argument, we can derive $\hat{x}^*_t(j,l^*)$ and $\hat{p}^*_t(j,l^*)$ for good j in location l^* of the foreign country:

$$\begin{aligned}
\hat{x}^*_t(j,l^*) &= \omega_j \hat{x}^*_{t-1}(j,l^*) + a_j \hat{\mu}^*_t + \frac{b_j}{1-\omega_j \rho L} \hat{\mu}^*_{t-1}, \\
\hat{p}^*_t(j,l^*) &= \lambda_j \hat{p}^*_{t-1}(j,l) - \lambda_j \hat{\mu}^*_t + (1-\lambda_j) \hat{x}^*_t(j,l^*).
\end{aligned}$$

The next proposition states that the good-level real exchange rates can show much richer short-run dynamics.

Proposition 2. *(The good-level real exchange rate dynamics under the dual stickiness model) Under the preference assumption $U(c,n) = \log c - \chi n$ and the assumption of money growth* (27) *and* (28)*, the stochastic process governing the good-level real exchange rate between any locations l and l^*, $\hat{q}_t(j,l,l^*)$ is of the form:*

$$\hat{q}_t(j,l,l^*) = \sum_{r=1}^{4} \phi_{j,r} \hat{q}_{t-r}(j,l,l^*) + \sum_{r=0}^{2} \theta_{j,r} \eta_{t-r} \tag{45}$$

where

$$
\begin{aligned}
\phi_{j,1} &= \tilde{\phi}_{j,1} + \rho, \qquad \tilde{\phi}_{j,1} = \lambda_j + \omega_j + \omega_j \rho \\
\phi_{j,2} &= \tilde{\phi}_{j,2} - \tilde{\phi}_{j,1}\rho, \qquad \tilde{\phi}_{j,2} = -[\lambda_j \omega_j + (\lambda + \omega_j)\omega_j \rho] \\
\phi_{j,3} &= \tilde{\phi}_{j,3} - \tilde{\phi}_{j,2}\rho, \qquad \tilde{\phi}_{j,3} = \lambda_j \omega_j^2 \rho \\
\phi_{j,4} &= -\tilde{\phi}_{j,3}\rho \\
\theta_{j,0} &= \lambda_j - (1-\lambda_j)a_j \\
\theta_{j,1} &= -\lambda_j(\omega_j + \omega_j\rho) + (1-\lambda_j)(\omega_j \rho a_j - b_j) \\
\theta_{j,2} &= \lambda_j \omega_j^2 \rho.
\end{aligned}
$$

In other words, the good-level real exchange rate follows an ARMA(4,2) process.

Proof. See Appendix B. □

This proposition further generalizes Proposition 1. When $\omega_j = 0$, we can obtain the same parameterization as (35).[11] Moreover, Appendix B shows that the SAR in this generalized case is given by

$$
\alpha_j = \sum_{r=1}^{4} \phi_{j,r} = 1 - (1-\lambda_j)(1-\omega_j)(1-\omega_j\rho)(1-\rho),
$$

Clearly, the slower the speed of information adjustment is, the larger the SAR becomes.

For a general ARMA process without any parameter restriction, it is not conventional to use the SAR as a measure of persistence, because of the presence of MA terms. However, if our model is correctly specified, we can show that both the long-run impact of cumulative impulse response of a unit monetary shock on real exchange rates and the SAR are strictly increasing function of λ_j, ω_j, and ρ. Furthermore, using the SAR is also convenient in computation and for the purpose of making comparison with simpler models introduced in the previous subsection. For these reasons, we focus on using the SAR as an approximate measure of persistence under the assumption that the process (45) is correctly specified.

[11] In particular, we obtain $\phi_{j,1} = \lambda_j + \rho$, $\phi_{j,2} = -\lambda_j\rho$, and $\phi_{j,3} = \phi_{j,4} = 0$ for the AR parameters and $\theta_{j,0} = \theta_j$ and $\theta_{j,1} = \theta_{j,2} = 0$ for the MA parameters.

The dual stickiness model works quite well in generating the persistence of a good-level real exchange rate, The left panel of Fig.3 shows the SAR among different ω_j's. The persistence is increasing in ω_j and is very high regardless of the infrequency of price changes.[12]

The left panel of Fig.4 plots the persistence over different λ_j's. This panel compares two extreme values of ω_j. One is the case in which firms producing good j updates their information every month. (i.e., $\omega_j = 0$.) The other is also an extreme case in which firms updates information every 50 months (implied by $\omega_j = 0.98$). Although DKT used the aggregate U.S. inflation data to estimate the average information delay to be approximately 7 months, we use the two extreme values for an ω_j because information stickiness may differ in terms of good's specifications. Interestingly, the persistence measure is very close to one whether prices are sticky or flexible in the latter extreme case.

Then, how well does the dual stickiness model account for the volatility of good-level real exchange rates? We again calculate a new scaling factor $f_3(\omega_j, \lambda_j, \rho, \beta)$ for this ARMA(4,2) process to evaluate the predicted volatility of good-level real exchange rates. We can write the predicted standard deviation of $\hat{q}_t(j, l, l^*)$ as $f_3(\omega_j, \lambda_j, \rho, \beta)\text{std}(\Delta S_t)$. The right panel of Fig.3 plots the scaling factor and suggests that the volatility grows exponentially as ω_j increases. The right panel of Fig.4 compares the scaling factors under zero and 50 month average information delays. The levels of the scaling factor are strikingly different between the two cases. They suggest that the volatility becomes substantially higher when the information adjustment is slower. Thus, unlike the case of a positive ρ, the introduction of information stickiness enhances the real exchange rate volatility to a large extent.

We summarize our theoretical findings on the effect of information stickiness as follows. Our theoretical assessments suggest that both the persistence and volatility of good-level real exchange rates predicted by the dual stickiness model can be quite large. It is true even if the price adjustment is relatively fast. Hence, we conjecture that adding the information stickiness into the Calvo model may explain the observed persistence and volatility of real exchange rates.

[12] Even if $\omega_j = 0$, $\hat{q}_t(j, l, l^*)$ has been already somewhat persistent. It is because of the effect of the AR(1) money growth.

3 Empirical Implementation

3.1 Data

The data source of our cross-border inter-city retail price deviations is the *Worldwide Cost of Living Survey* compiled by the Economist Intelligence Unit (EIU). It is an extensive annual survey of international retail prices that was originally designed to help managers to determine compensation levels of their employees residing in different cities of the world. The coverage of goods and services is broad enough to overlap significantly with what appears in a typical urban consumption basket (see Rogers (2007), for more detail on the comparison between EIU data and the CPI data from national statistical agencies). A notable advantage of the EIU data is the fact that all the individual good prices are listed in absolute terms with the survey conducted by a single agency in a consistent manner over time. Because of this convenient panel data format, a number of recent studies on international price dynamics have used this data, including Crucini and Shintani (2007), Engel and Rogers (2004), Parsley and Wei (2007) and Rogers (2007).

For a limited number of countries, the EIU data contains observations from multiple cities. In our empirical analysis, we focus on U.S.-Canadian city pairs since the assumption of the common probability of price adjustment for each good seems to be a reasonable approximation between the two neighboring countries.[13] After removing missing observations to construct a balanced panel for the period from 1990 to 2005, three cities out of 16 U.S. cities available in the survey are dropped, which resulted in a total of 52 city pairs consists of all possible pairs between the groups of 13 U.S. cities and 4 Canadian cities. The cities and categories of goods included in the analysis are shown in Fig. 5 and Table A1, respectively.

For each good j, the log of $q_t(j, l, l^*)$ for each year t $(= 1, ..., 16)$ is computed using the price level in a U.S. city l $(= 1, ..., 13)$ expressed in the U.S. dollar $(P_t(j, l))$, the price level in a Canadian city l^* $(= 1, ..., 4)$ expressed in the Canadian dollar $(P_t^*(j, l^*))$, and the spot U.S./Canadian dollar exchange rate (S_t), all from the EIU data. Since the resulting log real exchange rates represent the log deviations of the price in a Canadian city relative to that

[13] Alternatively, one may use the average of price change frequencies between the two countries, an approach employed in Kehoe and Midrigan (2007), when data from both countries are available.

of a U.S. city both expressed in a common currency, a negative value for the pair of Toronto and New York, for example, implies that the good is more expensive in New York than in Toronto at year t.

Next, for the frequency of price changes of individual goods, we utilize the numbers provided in the existing studies on the BLS (Bureau of Labor Statistics) data. Bils and Klenow (2004) used the BLS Commodities and Services Substitution Rate Table for 1995-1997 which contains the average frequencies of price changes of individual goods and services used for the U.S. CPI construction. Since Bils and Klenow's (2004) coverage period of 1995-1997 is a subset of the sample period of 1990-2005 in our EIU data set, we utilize the monthly average frequency of price changes, f_j , from Table A1 of their paper and transform it into the price stickiness parameter $\lambda_j = 1 - f_j$ for each good j.[14] We then match the EIU goods used to compute the real exchange rates to the BLS goods with the price stickiness information. Some goods are dropped in this matching process and final number of total goods becomes 165.

In a recent study by Nakamura and Steinsson (2007), Bils and Klenow's finding of fast price adjustment was revisited by using more detailed and updated BLS data. Using the CPI Research Database created by BLS, they defined the regular price change based on the removal of the temporary price changes caused by sales and found that the the median duration between regular price changes was 8 - 11 months depending on the treatment of substitutions. While we mainly use Bils and Klenow's (2004) frequency of price change data in our analysis, we additionally use Nakamura and Steinsson's (2007) data on the frequency of price changes from the period of 1998-2005 for the robustness check.

For the nominal exchange rate changes required for the theoretical volatility calculation, we use monthly changes in the log of the end-of-month U.S.-Canadian dollar spot rates. While both price stickiness parameter (frequency of price changes) and nominal exchange rates are available in monthly series, real exchange rates are only observed annually. The

[14]In some country which experienced a structural shift in inflation, an assumption of constant frequency of price changes over years may not be satisfied. For example, Ahlin and Shintani (2007) use Mexican price data on 44 goods and report that the average monthly frequency of price changes was 28% in 1994 and as large as 50% in 1995. We expect that this issue is less serious in our case since both U.S. and Canada had a stable inflation during the period under consideration.

small number of time series observation in a low frequency is the major limitation of the EIU data. However, thanks to a special dynamic feature of the theoretical model, the main implication of previously introduced propositions can be investigated even if only a short panel of annual data is available. In the following subsections, we will discuss in detail how to reconcile the mixed frequencies of observation in the dynamic panel estimation.

3.2 Reconciling monthly models with annual data

This subsection shows how we transform a monthly model into the one which have non-zero AR coefficients for multiples of 12 month lags and finite MA terms but have the remaining AR coefficients of zero. The transformation leads us to estimate the model via the annual data.

The easiest model is the Calvo model with $\rho = 0$. In this case, (35) directly implies that

$$\hat{q}_t(j, l, l^*) = \lambda_j \hat{q}_{t-1}(j, l, l^*) + \lambda_j \eta_t.$$

Clearly, by repeated substitutions, we get

$$\hat{q}_t(j, l, l^*) = \lambda_j^{12} \hat{q}_{t-12}(j, l, l^*) + \lambda_j \Lambda_j(L) \eta_t$$

where $\Lambda_j(L) = \sum_{r=0}^{11} \lambda_j^r L^r$. In this equation, the AR term is only a twelve month lag and the MA terms have a finite order of 11. This ARMA(12,11) model is simply equivalent to AR(1) in terms of annually sampled data since $\lambda_j \Lambda_j(L)\eta_t$ and $\hat{q}_{t-12}(j, l, l^*)$ are not correlated.

Such a transformation is not generally possible with a general ARMA process including AR(2) and ARMA(4,2) processes. However, below we show that it *is* possible to make such a transformation under our extended models.

The Calvo model ($\rho \neq 0$) First, we can rewrite the first order difference equation (36) as

$$\hat{q}_t(j, l, l^*) = \frac{\theta_j \Lambda_j(L)}{1 - \lambda_j^{12} L^{12}} \Delta \hat{S}_t, \tag{46}$$

Second, since $\Delta \hat{S}_t = \hat{\mu}_t - \hat{\mu}_t^*$, it immediately follows that

$$\Delta \hat{S}_t = \rho \Delta \hat{S}_{t-1} + \eta_t = \frac{R(L)}{1 - \rho^{12} L^{12}} \eta_t, \tag{47}$$

where $R(L) = \sum_{r=0}^{11} \rho^r L^r$. Substituting (47) into (46) yields:

$$\hat{q}_t(j,l,l^*) = (\lambda_j^{12} + \rho^{12})\hat{q}_{t-12}(j,l,l^*) - \lambda_j^{12}\rho^{12}\hat{q}_{t-24}(j,l,l^*) + \theta_j \Lambda_j(L) R(L)\eta_t, \tag{48}$$

which implies ARMA(24,22). Once again, the AR parameters are non-zero only if the lags are multiples of 12. Moreover, the MA terms are finite of 22 in this specific ARMA process.[15] Intuitively, this transformation is made possible because $\hat{q}_t(j,l,l^*)$ is the first order difference equation and the driving force $\Delta \hat{S}_t$ follows an AR(1) process. Conveniently, this monthly ARMA(24,22) becomes ARMA(2,1) in terms of annually sampled data.

The dual stickiness model A similar transformation is also possible in the dual stickiness model. The next proposition summarizes the transformation result.

Proposition 3. *In the dual stickiness model with ρ, λ_j, and $\omega_j \in (0,1)$, the ARMA(4,2) process characterized by* (45) *has an equivalent expression of the following ARMA(48,46) process:*

$$\hat{q}_t(j,l,l^*) = \sum_{r=1}^{4} \Phi_{j,12r}\hat{q}_{t-12r}(j,l,l^*) + \Theta_j(L)\eta_t, \tag{49}$$

where

$$\begin{aligned}
\Phi_{j,12} &= \tilde{\Phi}_{j,12} + \rho^{12}, \qquad \tilde{\Phi}_{j,12} = \lambda_j^{12} + \omega_j^{12} + (\omega_j\rho)^{12} \\
\Phi_{j,24} &= \tilde{\Phi}_{j,24} - \tilde{\Phi}_{j,12}\rho^{12}, \qquad \tilde{\Phi}_{j,24} = -[\lambda_j^{12}\omega_j^{12} + (\lambda_j^{12} + \omega_j^{12})\omega_j^{12}\rho^{12}] \\
\Phi_{j,36} &= \tilde{\Phi}_{j,36} - \tilde{\Phi}_{j,24}\rho^{12}, \qquad \tilde{\Phi}_{j,36} = \lambda_j^{12}\omega_j^{24}\rho^{12} \\
\Phi_{j,48} &= -\tilde{\Phi}_{j,36}\rho^{12} \\
\Theta_j(L) &= \Big\{(1 - \omega_j^{12}L^{12})(1 - (\omega_j^{12}\rho)^{12}L^{12})\lambda_j\Lambda_j(L)R(L) \\
&\quad -(1-\lambda_j)\Lambda_j(L)\Omega_j(L)R(L)\left[(1 - (\omega_j\rho)^{12}L^{12})a_j + b_j L(1 + \Omega_j^R(L))\right]\Big\} \\
\Omega_j(L) &= \sum_{r=0}^{11}\omega_j^r L^r, \quad \Omega_j^R(L) = \sum_{r=1}^{11}(\omega_j\rho)^r L^r.
\end{aligned}$$

Proof. See Appendix C. □

[15] It is because both $\Lambda_j(L)$ and $R(L)$ have the power of L of 11 in maximum.

The implications of Proposition 3 are as follows. First, the number of AR parameters are limited to four and these four parameters are the coefficients for 12, 24, 36, and 48 month lags. Thus, AR part has a form of autoregression on the past values of the real exchange rates in annual frequency. Second, if the AR part has the restriction described above and if the maximum order of MA coefficients is 46, the dual stickiness model with ρ, λ_j, and $\omega_j \in (0,1)$ can be written only with this representation. Third, this ARMA(48,46) process becomes ARMA(4,3) in terms of annually sampled data. Finally, under the representation, Appendix C also shows that the SAR is given by

$$\alpha_j = \sum_{r=1}^{4} \Phi_{j,12r} = 1 - (1-\rho^{12})(1-\lambda_j^{12})(1-\omega_j^{12})(1-(\omega_j\rho)^{12}), \tag{50}$$

which is again increasing in λ_j, ω_j and ρ.

3.3 Estimation

In this subsection, we describe the procedure to estimate the time series model transformed in the previous subsection using annual panel data. First, we adopt the following new notation to simplify the description. Previously, l and l^* were used for the domestic and foreign cities, respectively. Here, they are replaced by a new single index i $(= 1, ..., N)$ which represents each pair from two countries. Since the number of U.S. and Canadian cities used in the analysis is 13 and 4, respectively, the total number of cross-border city pairs is given by $N = 52$. In addition, the sampling frequency for the model was assumed to be monthly. With some abuse of notation, our new time subscript now represents the time in annual frequency. Namely, if the true data process is generated for each month $t^* = 1, ..., T^*$, we now only observe the series annually at the months of $t = 12 \times t^* = 1, ..., T(= T^*/12)$. With this newly introduced index, we define q_{it}^j as the *log* of the real exchange rate for good j between the city pair i at year t:

$$q_{it}^j = \ln q_t(j, l, l^*).$$

Thus, the former log deviation from the steady state $\widehat{q}_t(j, l, l^*)$ can be rewritten as $q_{it}^j - q_i^j$, where q_i^j is the long-run value which the Appendix D derives:

$$q_i^j = \ln q(j, l, l^*) = \ln \frac{[1 + \kappa^{1-\theta}(1 + \tau(j, l^*))^{1-\theta}]^{\frac{1}{1-\theta}}}{[1 + \kappa^{1-\theta}(1 + \tau(j, l))^{1-\theta}]^{\frac{1}{1-\theta}}}.$$

Intuitively, if the transportation cost exporting good j to l^* is high relative to that exporting the same type of good to l, q_i^j is positive. For example, if the transportation cost of exporting a good to Vancouver is relatively higher in that of exporting the same good to New York, the log of the real exchange rate of that good exceeds zero even in the long-run. if the transportation cost of exporting the same good to Toronto is relatively lower in that of exporting the same good to New York, q_i^j is below zero. Such heterogeneous long-run deviations justify the presence of the individual effect (the time invariant city pair-specific effect) in a panel estimation.

Second, in our model, the dynamic properties of q_{it}^j, including each AR coefficient $\Phi_{j,r}$ and the SAR $\alpha_j = \sum_{r=1}^{m} \Phi_{j,r}$, where m is the maximum order of AR coefficient, are all good specific. However, since all the time series models will be estimated separately for each good, using the panel data with many observation of city pairs, we will also temporarily drop both subscript and superscript j in the following description of the estimation procedure.

Based on the discussion on the annual transformation in the previous subsection, all the dynamics of the real exchange rate for a single good (with the symbol j dropped) can be written as

$$q_{it} = \sum_{r=1}^{m} \Phi_r q_{i,t-r} + \zeta_i + u_t + v_{it},$$

where ζ_i is the time invariant unobserved city pair-specific effect which allows long-run price difference between two cities, u_t is the common time effect which represents the exchange rate shocks and v_{it} is a residual term which represents the remainder component in good specific real exchange rate. This model format nests all the models under consideration in our paper: (i) the Calvo model with $\rho = 0$ implies $m = 1$; (ii) the Calvo model with $\rho \neq 0$ implies $m = 2$; and (iii) the dual stickiness model implies $m = 4$. For the individual specific effect ζ_i, we can easily see its relationship to the long-run mean and the persistence from $q_i = \zeta_i/(1 - \alpha)$ where $\alpha = \sum_{r=1}^{m} \Phi_r$. For the common time effect u_t, the Calvo model with $\rho \neq 0$ predicts a serial correlation of order one, while the dual stickiness model predicts a serial correlation of order three. However, in a short panel asymptotic with finite T, the common time effect can be treated as unknown parameters to be estimated with time dummies. In addition, since our main interest is to estimate the persistence expressed in terms of the SAR α, it is convenient to rewrite the model into a form often called as an augmented Dickey-Fuller

(ADF) format. Thus, the nested model is given by

$$q_{it} = \alpha q_{i,t-1} + \sum_{r=1}^{m-1} \gamma_r \Delta q_{i,t-r} + u' \widetilde{D}_t + \zeta_i + v_{it},$$

where $\Delta q_{i,t-r} = q_{i,t-r} - q_{i,t-r-1}$, $\gamma_r = \sum_{s=r+1}^{m} \Phi_s$ for $r = 1, ..., k-1$, $u = (u_{m+1}, ..., u_T)'$ is a vector of constants, $\widetilde{D}_t$ is a $(T-m) \times 1$ time dummy vector with one in the t-th position and zero otherwise.

To estimate this short dynamic panel model, we employ the generalized method of moments (GMM) estimator in the first differenced form for the purpose of eliminating the individual effect ζ_i. We follow Arellano and Bond (1991) in the choice of instruments and initial weighting matrix. In particular, the moment condition is given by

$$\mathbb{E}\left[q_{is} \left(\Delta q_{it} - \alpha \Delta q_{i,t-1} - \sum_{r=1}^{m-1} \gamma_r \Delta^2 q_{i,t-r} - \delta' D_t \right) \right] = 0$$

for $s = 1, ..., t-m-1$ and $t = m+2, ..., T$, where $\Delta^2 q_{i,t-r} = \Delta q_{i,t-r} - \Delta q_{i,t-r-1}$ $\delta = (\Delta u_{m+2}, ..., \Delta u_T)'$ is a vector of constants, D_t is a $(T-m-1) \times 1$ time dummy vector with one in the t-th position and zero otherwise. The total number of parameters to be estimated is $T-1$ with the number of moment conditions given by $(T-m)(T-m-1)/2$. Therefore, for the model to be (over-) identified, at least $T = 4$ is required for $m = 1$, $T = 6$ is required for $m = 2$, and $T = 9$ is required for $m = 4$. Since $T = 16$ is available in our sample, the number of over-identifying restrictions is 51, 76, and 90, respectively, for $m = 1, 2$, and 4. This GMM estimator for α is consistent under large N fixed T asymptotics.

4 Results

4.1 Persistence

In this subsection, we evaluate the performance of the sticky price model and its extension in explaining the observed persistence of the real exchange rate for each good j. Following the theoretical analysis, our empirical persistence measure is the SAR α_j.

We first revisit the benchmark model of Kehoe and Midrigan (2007) with an assumption of an iid money growth ($\rho = 0$) in the Calvo model. In this case, the theory predicts an AR(1) model and thus α_j is simply an AR(1) coefficient. A GMM estimation of α_j using annual U.S.-Canadian city pairs data yields a median of 0.56.[16] In terms of monthly frequency, our value corresponds to $0.56^{12} = 0.95$, which is slightly less than 0.98, the median value obtained by Kehoe and Midrigan (2007) based on bilateral real exchange rates of 66 goods between the U.S. and four European countries, Austria, Belgium, France and Spain.

The first panel in Fig.6 plots the estimated persistence measure α_j against the infrequency of the price adjustment in the annual frequency $\lambda_j^{12} = (1 - f_j)^{12}$ computed based on f_j from Bils and Klenow's (2004) table. A cross-sectional regression of α_j on λ_j^{12} yielded a significantly positive estimate of 0.30 (with a standard error of 0.08) which is consistent with the theoretical prediction at least in direction: more price stickiness implies higher persistence. However, 160 out of 165 goods lie above the 45 degree line ($\alpha_j = \lambda_j^{12}$) in the scatter plot with the regression slope being significantly less than unity. If the model performance is evaluated by computing the ratio of the predicted persistence (on the 45 degree line) to the observed persistence for each good, the model can explain merely a 6 percent of the total persistence for the median good. This confirms Kehoe and Midrigan's claim that a simple model of price stickiness alone is quantitatively insufficient to reproduce the observed persistence in good-level real exchange rates.

We next consider the effect of introducing serially correlated money growth ($\rho = 0.83$) in the Calvo model. On the whole, the persistence estimate α_j remains almost unchanged with a median value of 0.57 based on the AR(2) model. The regression slope shown in the second panel of Fig.6 is 0.35 and is again significantly positive. Recall that for a given λ_j, α_j is a monotonically increasing function of ρ (see the left panel of Fig.1). To be more specific, in annual frequency, the predicted SAR is given by

$$\alpha_j = 1 - (1 - \rho^{12})(1 - \lambda_j^{12}) = \lambda_j^{12} + \rho^{12} - \lambda_j^{12}\rho^{12}$$

and the effect of increasing ρ can be seen in the median value of the ratio of prediction and data provided in the upper panel of Table 1. In terms of the median, the theoretical persistence becomes the observed persistence when ρ is around 0.95. However, this value

[16]This value lies between the medians for OECD city pairs (0.65) and LDC city pairs (0.51) obtained by Crucini and Shintani (2007) based on the same data source.

is much higher than $\rho = 0.83$, the reference value based on CKM. Indeed, when $\rho = 0.83$ is used, only 31 percent of the persistence can be explained by the model (the number is provided as the first entry of the lower panel). This fact of an insufficient persistence of the money growth in explaining the persistence of real exchange rates can be also seen from the scatter plot. Recall that, from the left panel of Fig.2, increasing ρ shifts the theoretical line upward with a flatter slope. A similar theoretical prediction line with $\rho = 0.83$, expressed in the annual frequency basis, is also drawn in the second panel of Fig.6.[17] Compared to the 45 degree line in the first panel of the same figure ($\rho = 0$), the predicted line now becomes flatter but is still much steeper than the regression line. Indeed, about 95 percent of data points are still above the $\rho = 0.83$ line. Thus, again, the Calvo model is not very successful in explaining the persistence with a reasonable choice of money growth process.

Third, we now look at the role of information delay in explaining α_j under the framework of dual stickiness. To simplify the argument, here we assume the information delay parameter to be common across all the goods (namely, $\omega_j = \omega$ for all j). The persistence estimates based on the AR(4) model become somewhat lower with a median value of 0.51, but still are much higher than the level predicted by the standard Calvo model with no information delay (which corresponds to the $\omega = 0$ line shown in the lower panel of Fig.6). Recall that from the left panel of Fig.3, for a fixed value of λ_j and $\rho(= 0.83)$, α_j is strictly increasing in ω. This pattern is preserved in the SAR expressed in annual frequency through

$$\alpha_j = 1 - (1 - \rho^{12})(1 - \lambda_j^{12})(1 - \omega^{12})(1 - (\omega\rho)^{12}).$$

Based on this relationship, median of the ratio of theoretical value to observed value, provided in the lower panel of Table 1, increases along ω and reaches one at $\omega = 0.93$ which corresponds to 14 months of average duration between information updates. Therefore, at least in terms of the median, the dual stickiness model with a reasonable money growth process is capable of replicating the observed persistence. In the lower panel of Fig.6, shaded triangle area shows the range between the lowest predicted line with no information delay ($\omega = 0$) and the possibly highest predicted line with $\omega = 0.98$ which corresponds to a hypothetical maximum average duration between information updates of 50 months. Inter-

[17]The intercept of the theoretical line is $\rho^{12} = 0.83^{12} = 0.11$.

estingly, the regression line is located almost in the middle of the triangle with a slope of 0.56 which lies strictly between the slopes of the upper and lower bound prediction lines.

We now turn to the results based on f_j from Nakamura and Steinsson's (2007) data. Fig.7 shows the scatter plots of the pairs of $(\alpha_j, \lambda_j^{12})$ for (i) Calvo model with $\rho = 0$, (ii) Calvo model with $\rho \neq 0$, and (iii) dual stickiness model, respectively. For many goods, the removal of sales results in the lower value of f_j. Less frequent price changes increase the value of $\lambda_j^{12} = (1 - f_j)^{12}$, and makes most of the data point in the scatter plot shift toward right.[18] For all the models, the predicted persistence will be higher for the larger values of λ_j^{12}, and thus excluding the sales from frequency of price changes works in favor of the sticky price explanation of real exchange rate persistence. The proportion of the data points lie below the theoretical prediction line increased from 3 percent to 16 percent for the Calvo model with $\rho = 0$, and from 5 percent to 24 percent for the Calvo model with $\rho = 0.83$. For all the case, regression slopes shown in the scatter plots are again significant and positive, and the regression fit in terms of the coefficient of determination becomes uniformly better.[19] However, because of the rightward shift, more data points in the last panel of the figure fall outside the shaded triangle region representing the theoretical prediction of dual stickiness models.

We can also see the improvement in the ratio of predicted to the observed persistence provided in Table 2. As shown in the first entry of the upper panel of Table 2, even a benchmark Calvo model with $\rho = 0$ can account for 48 percent of the observed persistence, in comparison to 6 percent based on Bils and Klenow's frequency. The ratio increases as ρ increases, but because of the higher initial ratio, it becomes one at around $\rho = 0.92$ a value lower than previously selected value of $\rho = 0.95$. This newly selected ρ, however, is again higher than the CKM's reference value of $\rho = 0.83$. Since the ratio is 66 percent at $\rho = 0.83$ in the Calvo model, there is still a room for the information delay structure to fill the gap between the theoretical and observed value. The lower panel of Table 2 shows the effect of

[18]Note that α_j for each good j for each AR model remains unchanged between Figures 6 and 7. In addition, because the sample periods differ between the two data sets, this rightward shift may not be true for some goods.

[19]Regression coefficients are 0.36, 0.31, and 0.43 for each panel, respectively. Coefficients of determination increase from 8 to 26 percent, 10 to 17 percent, and 12 to 15 percent, respectively.

increasing ω on the prediction ratio based on Nakamura and Steinsson's data. The table shows that the 100 percent of the persistence can be explained at about $\omega = 0.90$ which corresponds to 9.5 months of average duration between information updates. This implies that the role of information stickiness does not need to be as strong as before.

So far, point estimates of α_j are used to evaluate the persistence in the data without taking account of estimation uncertainty. To incorporate the effect of estimation error in the analysis, we conduct the following exercise. First, using the asymptotic standard error formula of the GMM estimator of α_j, we constructed a two-sided 95 percent confidence interval for each good j. Using Figures 6 and 7, we then count the number of goods with the theoretical point of α_j predicted by λ_j^{12} being included in the confidence interval. On the one hand, with Bils and Klenow's data, the proportion of goods consistent with model prediction is 3 percent and 10 percent for the Calvo model with $\rho = 0$ and $\rho = 0.83$, respectively. This increases to 25 percent for the dual stickiness model with $\rho = 0.83$ and $\omega = 0.93$. On the other hand, with Nakamura and Steinsson data, the proportions for both Calvo models with $\rho = 0$ and $\rho = 0.83$ are 15 percent. In case of the dual stickiness model, confidence intervals included the predicted values for 18 percent of products. Furthermore, if we count the number of confidence intervals that contain a prediction from any of the dual stickiness models with ω between 0 (no information delay) and 0.98 (50 month delay on average), the same proportion increased to 90 percent for Bils and Klenow's data and 74 percent for Nakamura and Steinsson's data.

In summary, our evidence based on various evaluation method shows that observed persistence is well explained by adding an information stickiness feature in the basic Calvo sticky price model.

4.2 Volatility

The second puzzle brought up in Kehoe and Midrigan's (2007) study is on the observation of too much volatility in good-level real exchange rates which can neither be explained by a simple sticky price model nor a model with pricing complementarities. In this subsection, we focus on the evaluation of our models in terms of explaining observed volatility.

First, note that the variance of real exchange rate predicted by the model has the same implication to both annually sampled data and monthly sampled data. Therefore, unlike the measure of persistence, using annual real exchange rates rather than monthly rates requires no further computation and results provided in Section 2 is directly applicable. Second, however, using a pooled sample variance as a volatility measure is not appropriate since it includes the variance component due to the dispersion of long-run real exchange rate q_i^j among city pairs in our panel data. In addition, the theoretical model predicts volatility caused by the nominal exchange rate fluctuation which is common to all the products, but is not designed to incorporate the idiosyncratic variance component such as the one due to time-varying location specific shocks. For this reason, we conduct a variance decomposition based on a standard two-way error components model and employ the extracted variance component due to a time specific shocks as our measure of volatility. This volatility measure seems to be a reasonable choice in our study because it is consistent with the idea of using time dummies in the dynamic panel estimation to incorporate the common time specific shocks in our previous analysis of persistence. For the theoretical volatility level, we use the sample standard deviation of monthly nominal exchange rate growth multiplied by the scaling factor obtained from the theoretical model. The performance of the model is then evaluated by the ratio of the theoretical standard deviation to the observed standard deviation of common time specific component extracted from the data.

We start with looking at the results presented in Table 3 based on Bils and Klenow's data. The upper panel of the table shows the median of the ratio of the standard deviation predicted by the Calvo model to the observed standard deviation. The benchmark Calvo model with no serial correlation ($\rho = 0$) can explain only 13 percent of the variation in the data. Thus, the evidence of excess volatility discovered by Kehoe and Midrigan (2007) is also confirmed in our panel data of the U.S.-Canadian city pairs. Can we explain this observed volatility with an introduction of serially correlated money growth? Unfortunately, unlike the persistence, the predicted volatility is not a monotonically increasing function of ρ. Examples presented in the right panel of Fig.1 show that the volatility decreases monotonically for goods with small $\lambda_j = 1 - f_j$ and increases only in some range of ρ for goods with a larger λ_j. As a result of the combination of the two effects from many goods, none of the median ratio presented in the upper panel of Table 3 is above one and maximum value is only 15 percent at $\rho = 0.52$.

In contrast to the effect of ρ, the right panel of the Fig.4 shows that the volatility increases monotonically with ω in the dual stickiness model for any given values of λ_j and ρ. The lower panel of Table 3 presents the ratio of standard deviations based on the dual stickiness model with various ω's when the CKM's reference value of $\rho = 0.83$ is used for the money growth process. With an introduction of the information delay, the volatility can now be fully explained at $\omega = 0.94$ which implies 17 months of average duration between information updates. As shown in the previous section, the observed persistence can be reproduced if any value of ρ is allowed without introducing information stickiness. For the volatility, however, the observation can be replicated only under the framework of dual stickiness model. In this sense, the information delay plays an essential role in explaining the volatility.

We now turn Nakamura and Steinsson data with the effect of sales removed from f_j. The median of the ratio of the predicted standard deviation to the observed one for each model is shown in Table 4. The performance of the Calvo model, in terms of explaining volatility, clearly improves over the case of using Bils and Klenow's data. For example, the ratio increases from 13 percent to 23 percent when $\rho = 0$. Under this benchmark Calvo model, the real exchange rate follows an AR(1) process. Thus the scaling factor (standard deviation of q_{it}^j divided by the standard deviation of ΔS_t) is simply $\lambda_j/\sqrt{1-\lambda_j^2}$ and is increasing function of $\lambda_j = 1 - f_j$. Since the removal of sales results in the lower values of f_j, using Nakamura and Steinsson's data increases the theoretical volatility level. However, the degree of increased theoretical volatility is still insufficient to fully explain the observed volatility under the standard Calvo model. The maximum of the proportion of volatility which can be explained by the model is 43 percent at $\rho = 0.80$. Thus, the role of sticky information is again crucial in explaining the volatility of the good-level real exchange rates, even if we use Nakamura and Steinsson's data. The lower panel of Table 4 shows that 100 percent of observed volatility can be explained when $\omega = 0.92$ which implies 12 months of average duration between information updates. In comparison to the result from Bils and Klenow's data, the reduction of ω reflects the fact that a larger component in the variance is already explained by the reduction of price change frequency alone in the Nakamura and Steinsson's data.

4.3 Infrequency of information updating

In the previous subsections, we have shown that an introduction of information stickiness into the Calvo model can fully explain the medians of persistence and volatility by searching for the *common* average information delay. In this subsection, we will briefly evaluate the obtained common values of average information delay by comparing existing macro empirical studies on sticky information. Then, we relax our assumption of common information delay ($\omega_j = \omega$ for all j). That is, we consider *good-specific* average information delays which account for the individual persistence or volatility of good-level real exchange rates. This consideration allows us to infer the *distribution* of average information delays among goods. We will then assess our results by comparing micro studies on price reviews in the U.S.

To evaluate common ω estimates, we first compare them with previous studies' estimates on information stickiness based on the aggregate inflation. Using the aggregate data on inflation over 1960:Q1 - 2005:Q2, DKT find that information delay, on average, is 7.6 months with 95 percent confidence intervals between 5.6 and 25.4 months. Knotek (2006) introduces information stickiness into the fixed menu cost model and finds the average duration between information updates to be 20.4 months over 1983:Q1 - 2005:Q4. Thus, all of our common ω estimates are in line with previous estimates based on aggregate inflation.

The durations between information updates calculated from Bils and Klenow's price change frequency are 14 months from the persistence and 17 months from the volatility, respectively. Thus, they are relatively longer than DKT's point estimate of 7.6 months. Bils and Klenow (2004) price frequency data suggests that prices are substantially flexible. Hence, the firms must be inattentive enough to replicate the observed good-level real exchange rate dynamics.

Bils and Klenow (2004) argue that their micro evidence does not support the standard Calvo model of price stickiness and suggest that models where information plays an important role may better explain price dynamics. In this sense, we also compare our common values calculated from Bils and Klenow's data with estimates from empirical studies on the pure sticky information model rather than the dual stickiness model by DKT. For instance, Andrés, López-Salido and Nelson (2005) estimate the average information duration to be

20 months. Kahn and Zhu (2006) find that the point estimates of average duration range between 9 and 23 months. In sum, our common ω calculated from Bils and Klenow's data on the frequency of price changes is more or less consistent with existing studies' estimates on the sticky information model.

Turning to the values of common ω based on Nakamura and Steinssons data, the frequency of price changes implies the average information delay of 9.5 months from the SAR and 12 months from the volatility. These low values of ω reflect the fact that less frequent Nakamura and Steinsson's regular prices do not require that firms be much inattentive to the state of the economy to be consistent with the good-level real exchange rate dynamics. In any case, our obtained values of 9.5 months and 12 months calculated from their data seem consistent with DKT's confidence intervals.

So far, our common values of ω was obtained to match the persistence and volatility for the median good. We can also relax the assumption of common ω and allow for a *good specific* ω which explains the individual persistence or volatility of good-level real exchange rates. Good specific information delays from persistence and volatility can be obtained as follows.

First, to calculate good specific information delays from the persistence, we define the theoretical SAR $\alpha_j(\omega_j|\lambda_j, \rho) = 1 - (1-\rho^{12})(1-\lambda_j^{12})(1-\omega_j^{12})(1-(\omega_j\rho)^{12})$ for each good and construct the criterion function such that

$$\min_{\omega_j \in [0,1)} [\hat{\alpha}_j - \alpha_j(\omega_j|\lambda_j, \rho)]^2,$$

where $\hat{\alpha}_j$ denotes the GMM estimate obtained in the previous subsection. For the theoretical SAR, we take $\rho = 0.83$ from CKM and $\lambda_j = 1 - f_j$ from the frequency of price changes calculated by either Bils and Klenow (2004) or Nakamura and Steinsson (2007).[20]

Second, to calculate good specific information delays from the volatility, we again use the fact that the volatility of real exchange rates predicted by the model has the same implication

[20]Note that the criterion function may have a corner solution. As the left panel of Fig.3 shows, the SAR is strictly increasing in ω_j. Thus, when the GMM estimate $\hat{\alpha}_j$ is lower than any theoretical values of the SAR, ω_j must take a value of zero. Moreover, when $\hat{\alpha}_j$ is very large, ω_j takes a unreasonably large value such that the implied duration between information updates $1/(1-\omega_j)$ extremely long. In this case, we treat such a good as an outlier.

to both annually sampled and monthly sampled data. First, we calculate the theoretical prediction of the standard deviation of a good-level real exchange rate from ARMA(4,2) process. Given λ_j, ρ, and β, it is a function of ω_j. Second, for each of the goods, we use the standard deviation from our dataset to construct the following criterion function:

$$\min_{\omega_j \in [0,1)} [\mathrm{std}(q_{it}^j) - f_3(\omega_j, \lambda_j, \rho, \beta)\mathrm{std}(\Delta S_t)]^2,$$

where $\mathrm{std}(q_{it}^j)$ is the extracted standard deviation component due to a time specific shocks, while $f_3(\omega_j, \lambda_j, \rho, \beta)\mathrm{std}(\Delta S_t)$ is the predicted standard deviation from ARMA(4,2) process, given $\rho = 0.83$, $\beta = 0.99$ and $\lambda_j = 1 - f_j$ from either Bils and Klenow (2004) or Nakamura and Steinsson (2007).[21]

We now look at the distribution of good-specific average durations of information updates $1/(1-\omega_j)$ based on the frequency of price changes based on Bils and Klenow (2004). Fig.8 shows the relative histogram of information delays implied by the persistence and volatility of good-level real exchange rates. The line in each panel is the kernel density estimates of the distribution. On the whole, the shape of the distribution is similar as the two kernel density estimates suggest.[22] The median of the durations implied by persistence is 12.9 months while that of the durations implied by volatility is 16.6 months. These values are quite close to the average duration under the common ω assumption (14 months from persistence and 17 month from volatility). Interestingly, with the frequency of price changes based on Bils and Klenow (2004), the model can explain only 6.1 or 11.5 percent of goods without relying on information stickiness. The remaining goods need to have a positive good specific ω_j to fully explain good-level real exchange rate dynamics.

Next, we turn to Fig.9 which uses Nakamura and Steinsson's (2007) data on the frequency of price changes. Once again, the kernel density estimates suggest that the shapes of distribution are similar each other.[23] The median duration between information updates implied

[21] This criterion function may also have a corner solution when $\mathrm{std}(q_{it}^j)$ is smaller than the theoretical prediction. In this case, ω_j takes a value of zero.

[22] Descriptive statistics of the distribution are slightly different between the distributions implied by persistence and volatility. From the distribution implied by persistence, we obtain the standard deviation of 13.6, the skewness of 2.0, and the kurtosis of 5.7. On the other hand, we obtain the standard deviation of 15.6, the skewness of 1.5, and the kurtosis of 3.4 from the distribution implied by volatility.

[23] Descriptive statistics after trimming outliers are as follows. From the distribution implied by persistence,

by persistence is 8.2 months while that implied by volatility is 11.9 months. Thus, the information stickiness plays less important role than before. Moreover, unlike the distribution calculated from Bils and Klenow's data, more goods need not to have information stickiness. (33.3 percent from persistence and 21.8 percent from volatility.) Although relatively many goods need not to rely on information stickiness due to the exclusion of sales, approximately two-thirds of goods still need to have a positive ω_j to fully explain good-level real exchange rate dynamics.

Finally, we ask whether the obtained distributions are, on the whole, consistent with micro studies on prices. Unfortunately, no micro studies provide directly comparable distribution of differences in information among goods. However, survey results on price reviews made by firms may serve for our purpose. Fabiani, Druant, Hernando, Kwapil, Laudau, Loupias, Martins, Matha, Sabbatini, Stahl and Stokman (2005) argue that the frequency of price reviews rather than price changes "could be related to the arrival of information." According to Fabiani et. al. (2005), when additional information on the state of the economy infrequently arrives, it is sensible for firms to review prices infrequently. In this sense, we can exploit survey results for price reviews.

Blinder, Canetti, Lebow and Rudd (1998) surveyed U.S. firms about price setting behavior in the beginning of 1990s and their results for price reviews allow us to assess our distributions of average arrival of information. For price reviews, they received 121 responses out of 200 respondents about price reviews. They ask a customary time interval (e.g., daily, weekly, monthly, quarterly, and yearly) between price reviews for surveyed firms' most important product. Table 5 compares our distributions of durations of information updates with Blinder et. al. (1998) survey results. Overall, our distributions of duration between information updates seem to match the distribution of price reviews well. In particular, our results are quite close to their survey results when the frequency of price changes is taken from Nakamura and Steinsson (2007).

we obtain the standard deviation of 10.8, the skewness of 2.3, and the kurtosis of 9.1. From the distribution implied by volatility, the corresponding measures are 16.1, 1.9, and 4.7, respectively.

5 Conclusion

Using highly disaggregated price data from U.S. and Canadian cities, we have confirmed Kehoe and Midrigan's main finding that the standard Calvo-type sticky price model poorly explains the persistence and volatility of good-level real exchange rates. We found that this puzzling but stimulating result remains robust to a change from Bils and Klenow's data to Nakamura and Steinsson's data on the frequency of price changes. The robustness of their finding suggests that the Calvo sticky price model needs to be modified. We offered a possible solution to this puzzling result by extending the standard Calvo model such that only a fraction of firms have the up-to-date information when resetting prices. Due to the infrequent arrival of information, real exchange rates become more persistent and keep track of the volatile nominal exchange rate even if price adjustment is relatively fast. Our model can explain both persistence and volatility within a reasonable range of average information delay.

We have limited our attention to the implications of our model under many simplifying assumptions. Therefore, there are many promising avenues for future research. For example, what would happen to the prediction of our model if pricing complementarities are included? What would be the impact on good-level real exchange rate dynamics if the non-traded inputs in producing a good are included in the model?[24] What would happen to good-level real exchange rate dynamics if deterministic price setting schemes (e.g., the Taylor-type sticky price model) or deterministic information updating schemes is employed?[25] We believe that answering these questions would help us further understanding the anomaly.

[24]See Crucini, Telmer and Zachariadis (2005) for this line of research.

[25]See Dupor and Tsuruga (2005) for the impact of different information updating assumptions. They found that the pure sticky information model's predictions substantially differ between the random and deterministic information updating schemes.

A The closed form solution to $\hat{x}_t(j,l)$

To derive the closed form solution to $\hat{x}_t(j,l)$, we use the close form solution to $\hat{p}_{H,t}(j,l)$, given $\hat{\mu}_t$ follows AR(1). It has been already derived from (31) under an AR $\hat{\mu}_t$:

$$\hat{p}_{H,t}(j,l) = \left[\frac{\lambda_j\beta\rho}{1-\lambda_j\beta\rho}\right]\hat{\mu}_t,$$

which implies

$$\begin{aligned}\mathbb{E}_{t-k-1}[\Delta\hat{p}_{H,t}(j,l)] &= \left[\frac{\lambda_j\beta\rho}{1-\lambda_j\beta\rho}\right]\mathbb{E}_{t-k-1}(\hat{\mu}_t-\hat{\mu}_{t-1})\\ &= \left[\frac{\lambda_j\beta\rho}{1-\lambda_j\beta\rho}\right](\rho^{k+1}\hat{\mu}_{t-k-1}-\rho^k\hat{\mu}_{t-k-1}).\end{aligned}$$

Using this result, we can express $\hat{x}_t(j,l)$ as $\hat{x}_{t-1}(j,l)$ and $\{\hat{\mu}_{t-k}\}_{k=0}^{\infty}$:

$$\begin{aligned}\hat{x}_t(j,l) = \omega_j\hat{x}_{t-1}(j,l) - \omega_j\mu_t + (1-\omega_j)\left[\frac{\lambda_j\beta\rho}{1-\lambda_j\beta\rho}\right]\hat{\mu}_t\\ + \omega_j(1-\omega_j)\sum_{k=0}^{\infty}\omega_j^k\left[\frac{\lambda_j\beta\rho}{1-\lambda_j\beta\rho}\right](\rho^{k+1}\hat{\mu}_{t-k-1}-\rho^k\hat{\mu}_{t-k-1}+\hat{\mu}_{t-k-1}).\end{aligned}$$

Using a lag operator L, we can obtain

$$\begin{aligned}\hat{x}_t(j,l) = \omega_j\hat{x}_{t-1}(j,l) - \omega_j\hat{\mu}_t + (1-\omega_j)\frac{\lambda_j\beta\rho}{1-\lambda_j\beta\rho}\hat{\mu}_t\\ + \omega_j\frac{\lambda_j\beta\rho}{1-\lambda_j\beta\rho}(1-\omega_j)\sum_{k=0}^{\infty}\omega_j^k\rho^k L^k\left\{\left[\frac{\lambda_j\beta\rho}{1-\lambda_j\beta\rho}\right][\rho-1]+\rho\right\}\hat{\mu}_{t-1}.\end{aligned}$$

Using $\sum_{k=0}^{\infty}\omega_j^k\rho^k L^k = (1-\omega_j\rho L)^{-1}$ and arranging terms yields the close form solution to $\hat{x}_t(j,l)$ given by (43).

B The proof of proposition 2

To prove Proposition 2, we use AR(1) structures for $\hat{p}_t(j,l)$ and $\hat{\mu}_t$ and an AR(2,1) structure for $\hat{x}_t(j,l)$. We have

$$\begin{aligned}\hat{p}_t(j,l) &= \lambda_j\hat{p}_{t-1}(j,l) - \lambda_j\hat{\mu}_t + (1-\lambda_j)\hat{x}_t(j,l)\\ \hat{\mu}_t &= \rho\hat{\mu}_{t-1}+\varepsilon_t\\ \hat{x}_t(j,l) &= \omega_j\hat{x}_{t-1}(j,l) + a_j\hat{\mu}_t + \frac{b_j}{1-\omega_j\rho L}\hat{\mu}_{t-1}\end{aligned}$$

from (44), (27), and (43), respectively. We can rewrite the first and the third equations as follows:

$$\begin{aligned}\hat{p}_t(j,l) &= -\frac{\lambda_j}{1-\lambda_j L}\hat{\mu}_t + \frac{1-\lambda_j}{1-\lambda_j L}\hat{x}_t(j,l)\\ \hat{x}_t(j,l) &= \frac{a_j}{1-\omega_j L}\hat{\mu}_t + \frac{b_j}{(1-\omega_j L)(1-\omega_j\rho L)}\hat{\mu}_{t-1}.\end{aligned}$$

We eliminate $\hat{x}_t(j,l)$ from these equations to get

$$\begin{aligned}(1-\lambda_j L)(1-\omega_j L)(1-\omega_j\rho L)\hat{p}_t(j,l) =&(1-\lambda_j)a_j(1-\omega_j\rho L)\hat{\mu}_t + (1-\lambda_j)b_j\hat{\mu}_{t-1}\\ &-\lambda_j(1-\omega_j L)(1-\omega_j\rho L)\hat{\mu}_t.\end{aligned}$$

Arranging terms of the right hand side of the equation yields

$$\begin{aligned}(1-\lambda_j L)(1-\omega_j L)(1-\omega_j\rho L)\hat{p}_t(j,l) =&-\left[\lambda_j - (1-\lambda_j)a_j\right]\hat{\mu}_t\\ &+\left[\lambda_j(\omega_j+\omega_j\rho) - (1-\lambda_j)(\omega_j\rho a_j - b_j)\right]\hat{\mu}_{t-1}\\ &-\lambda_j\omega_j^2\rho\hat{\mu}_{t-2}.\end{aligned}$$

Using the definition of $\theta_{j,0}$, $\theta_{j,1}$ and $\theta_{j,2}$ defined in Proposition 2, we get

$$(1-\lambda_j L)(1-\omega_j L)(1-\omega_j\rho L)\hat{p}_t(j,l) = -\theta_{j,0}\hat{\mu}_t - \theta_{j,1}\hat{\mu}_{t-1} - \theta_{j,2}\hat{\mu}_{t-2}.$$

The left hand of the equation can be extended so that

$$(1-\tilde{\phi}_{j,1}L - \tilde{\phi}_{j,2}L^2 - \tilde{\phi}_{j,3}L^3)\hat{p}_t(j,l) = -\theta_{j,0}\hat{\mu}_t - \theta_{j,1}\hat{\mu}_{t-1} - \theta_{j,2}\hat{\mu}_{t-2}.$$

Since the money growth rate follows an AR(1), $\hat{\mu}_t = (1-\rho L)^{-1}\varepsilon_t$. Then,

$$(1-\rho L)(1-\tilde{\phi}_{j,1}L - \tilde{\phi}_{j,2}L^2 - \tilde{\phi}_{j,3}L^3)\hat{p}_t(j,l) = -\theta_{j,0}\varepsilon_t - \theta_{j,1}\varepsilon_{t-1} - \theta_{j,2}\varepsilon_{t-2}.$$

Arranging terms the left hand of the equation gives $\phi_{j,1}$, $\phi_{j,2}$, $\phi_{j,3}$, and $\phi_{j,4}$:

$$\hat{p}_t(j,l) = \sum_{r=1}^{4}\phi_{j,r}\hat{p}_t(j,l) - \sum_{r=0}^{2}\theta_{j,r}\varepsilon_{t-r}.$$

By the similar argument, we can derive the price index for good j of location l^*:

$$\hat{p}_t^*(j,l^*) = \sum_{r=1}^{4}\phi_{j,1}\hat{p}_t^*(j,l^*) - \sum_{r=0}^{2}\theta_{j,r}\varepsilon_{t-r}^*.$$

Because $\hat{q}_t(j,l,l^*) = \hat{p}_t^*(j,l^*) - \hat{p}_t(j,l)$, we obtain (45).

Finally, note that the coefficient of $\hat{p}_t(j,l)$ is

$$(1-\rho L)(1-\tilde{\phi}_{j,1}L - \tilde{\phi}_{j,2}L^2 - \tilde{\phi}_{j,3}L^3) = (1-\rho L)(1-\lambda_j L)(1-\omega_j L)(1-\omega_j \rho L).$$

It implies that the SAR $\sum_{r=1}^4 \phi_{j,r}$ is equal to $1-(1-\rho)(1-\lambda_j)(1-\omega_j)(1-\omega_j\rho)$. Because the AR coefficients are the same between the same type of good j, it proves Proposition 2.

C Proof of proposition 3

To consider the transformation in the dual stickiness model, note that (43) can be rewritten as

$$\hat{x}_t(j,l) = \omega_j \hat{x}_{t-1}(j,l) + a_j \hat{\mu}_t + \frac{b_j L}{1-\omega_j \rho L}\hat{\mu}_t,$$

using a lag operator L. This equation has an infinite MA term because the third term of the right hand side has $(1-\omega_j\rho L)^{-1}\hat{\mu}_t$. We first work on this term.

The infinite MA form $(1-\omega_j\rho L)^{-1}\hat{\mu}_t$ is

$$\begin{aligned}
(1-\omega_j\rho L)^{-1}\hat{\mu}_t \qquad &= \hat{\mu}_t &&+ \sum_{r=1}^{11}(\omega_j\rho)^r \hat{\mu}_{t-r} \\
&+(\omega_j\rho)^{12}\hat{\mu}_{t-12} &&+ (\omega_j\rho)^{12}\sum_{r=1}^{11}(\omega_j\rho)^r \hat{\mu}_{t-r-12} \\
&+(\omega_j\rho)^{24}\hat{\mu}_{t-24} &&+ (\omega_j\rho)^{24}\sum_{r=1}^{11}(\omega_j\rho)^r \hat{\mu}_{t-r-24} + \cdots .
\end{aligned}$$

Collecting terms by columns yields

$$\begin{aligned}
(1-\omega_j\rho L)^{-1}\hat{\mu}_t &= (1+(\omega_j\rho)^{12}L^{12} + (\omega_j\rho)^{24}L^{24} + \cdots)\hat{\mu}_t \\
&\quad +(1+(\omega_j\rho)^{12}L^{12} + (\omega_j\rho)^{24}L^{24} + \cdots)\sum_{r=1}^{11}(\omega_j\rho)^r \hat{\mu}_{t-r} \\
&= \frac{1}{1-(\omega_j\rho)^{12}L^{12}}\hat{\mu}_t + \frac{1}{1-(\omega_j\rho)^{12}L^{12}}\sum_{r=1}^{11}(\omega_j\rho)^r L^r \hat{\mu}_t \\
&= \frac{1+\Omega_j^R(L)}{1-(\omega_j\rho)^{12}L^{12}}\hat{\mu}_t,
\end{aligned}$$

where $\Omega_j^R(L) = \sum_{r=1}^{11}(\omega_j\rho)^r L^r$.

Using this result, we obtain the first order difference equation for $\hat{x}_t(j,l)$:

$$\hat{x}_t(j,l) = \omega_j \hat{x}_{t-1}(j,l) + \left[a_j + \frac{b_j L(1+\Omega_j^R(L))}{1-(\omega_j\rho)^{12}L^{12}} \right] \hat{\mu}_t.$$

Equivalently, by repeated substitutions,

$$\hat{x}_t(j,l) = \omega_j^{12} \hat{x}_{t-12}(j,l) + \left[a_j + \frac{b_j L(1+\Omega_j^R(L))}{1-(\omega_j\rho)^{12}L^{12}} \right] \Omega_j(L)\hat{\mu}_t, \tag{51}$$

where $\Omega_j(L) = \sum_{r=0}^{11} \omega_j^r L^r$.

Similarly, the equation for the good j price index is the first order equation given by (44). It implies

$$\hat{p}_t(j,l) = \lambda_j^{12}\hat{p}_{t-12}(j,l) - \lambda_j\Lambda_j(L)\hat{\mu}_t + (1-\lambda_j)\Lambda_j(L)\hat{x}_t(j,l). \tag{52}$$

Substituting (51) into (52) yields

$$\begin{aligned} \hat{p}_t(j,l) =& -\frac{\lambda_j\Lambda_j(L)}{1-\lambda_j^{12}L^{12}}\hat{\mu}_t \\ &+ \frac{(1-\lambda_j)\Lambda_j(L)\Omega_j(L)\left[(1-(\omega_j\rho)^{12}L^{12})a_j + b_j L(1+\Omega_j^R(L))\right]}{(1-\lambda_j^{12})(1-\omega_j^{12}L^{12})(1-(\omega_j\rho)^{12}L^{12})}\hat{\mu}_t. \end{aligned} \tag{53}$$

Analogously, we can obtain a similar equation for $\hat{p}_t^*(j,l^*)$. Then, noting that $\hat{q}_t(j,l,l^*) = \hat{p}_t^*(j,l^*) - \hat{p}_t(j,l)$ and $\Delta\hat{S}_t = \hat{\mu}_t - \hat{\mu}_t^*$, we can obtain the following equation for the good-level real exchange rate:

$$\begin{aligned} \hat{q}_t(j,l,l^*) =& \frac{\lambda_j\Lambda_j(L)}{1-\lambda_j^{12}L^{12}}\Delta\hat{S}_t \\ &- \frac{(1-\lambda_j)\Lambda_j(L)\Omega_j(L)\left[(1-(\omega_j\rho)^{12}L^{12})a_j + b_j L(1+\Omega_j^R(L))\right]}{(1-\lambda_j^{12})(1-\omega_j^{12}L^{12})(1-(\omega_j\rho)^{12}L^{12})}\Delta\hat{S}_t. \end{aligned}$$

Arranging the terms yields

$$\begin{aligned} &(1-\lambda_j^{12}L^{12})(1-\omega_j^{12}L^{12})(1-(\omega_j\rho)^{12}L^{12})\hat{q}_t(j,l,l^*) \\ =&\Big\{(1-\omega^{12}L^{12})(1-(\omega_j\rho)^{12}L^{12})\lambda_j\Lambda_j(L)R(L) \\ &- (1-\lambda_j)\Lambda_j(L)\Omega_j(L)R(L)\left[(1-(\omega_j\rho)^{12}L^{12})a_j + b_j L(1+\Omega_j^R(L))\right]\Big\}\Delta\hat{S}_t. \end{aligned}$$

The terms inside the curly bracket gives $\Theta_j(L)$. Moreover, the first line of the terms has non-zero coefficient for L^{46}, because $(1-\omega^{12}L^{12})(1-(\omega_j\rho)^{12}L^{12})$ have a non-zero coefficient for L^{24} and $\Lambda_j(L)R(L)$ have a non-zero coefficient for L^{22}. Since the second line of the terms inside the curly brackets have L^{45}, the maximum power for L is 46.

From the argument in Appendix B, we can obtain, from the left hand side, $\tilde{\Phi}_{j,12}$, $\tilde{\Phi}_{j,24}$, $\tilde{\Phi}_{j,36}$. Finally, (47) implies

$$(1-\rho^{12}L^{12})(1-\tilde{\Phi}_{j,12}L^{12}-\tilde{\Phi}_{j,24}L^{24}-\tilde{\Phi}_{j,36}L^{36})\hat{q}_t(j,l,l^*)=\Theta_j(L)\eta_t,$$

which gives us $\Phi_{j,12}$, $\Phi_{j,24}$, $\Phi_{j,36}$ and $\Phi_{j,48}$.

The SAR can be easily obtained. Clearly,

$$\sum_{r=1}^{4}\Phi_{j,12r}=1-(1-\rho_j^{12})(1-\lambda_j^{12})(1-\omega_j^{12})(1-(\omega_j\rho)^{12}).$$

D The long-run value of a good-level real exchange rate

This appendix shows the long-run value of $q_t(j,l,l^*)$. In what follows, we use variables without time subscript to denote the steady state value.

Consider the steady state value of the price of good j in location l. In the steady state, firms in the home country set prices such that

$$P_H(j,l)=\frac{\theta}{\theta-1}W=\chi\frac{\theta}{\theta-1}M.$$

Here, we used (18). Firms in the foreign country choose prices such that

$$P_F(j,l)=\frac{\theta}{\theta-1}(1+\tau(j,l))SW^*=\chi\frac{\theta}{\theta-1}\kappa(1+\tau(j,l))M.$$

because of (17) and (18). Therefore, the price of good j in location l is

$$P(j,l)=\frac{\chi}{2}\frac{\theta}{\theta-1}[1+\kappa^{1-\theta}(1+\tau(j,l))^{1-\theta}]^{\frac{1}{1-\theta}}M. \tag{54}$$

By similar argument, we can derive $P^*(j,l^*)$ as follows:

$$P^*(j,l^*)=\frac{\chi}{2}\frac{\theta}{\theta-1}[\kappa^{-(1-\theta)}+(1+\tau(j,l))^{1-\theta}]^{\frac{1}{1-\theta}}M^*. \tag{55}$$

Given the good-level real exchange rate for good j of a location pair between l and l^* is given by $q(j, l, l^*) = SP(j, l)/P(j, l^*)$, the equations (17), (54), and (55) imply

$$q(j, l, l^*) = \frac{[1 + \kappa^{1-\theta}(1 + \tau(j, l^*))^{1-\theta}]^{\frac{1}{1-\theta}}}{[1 + \kappa^{1-\theta}(1 + \tau(j, l))^{1-\theta}]^{\frac{1}{1-\theta}}}. \tag{56}$$

Thus, the long-run value of a good-level of real exchange rate depends on the location pair.

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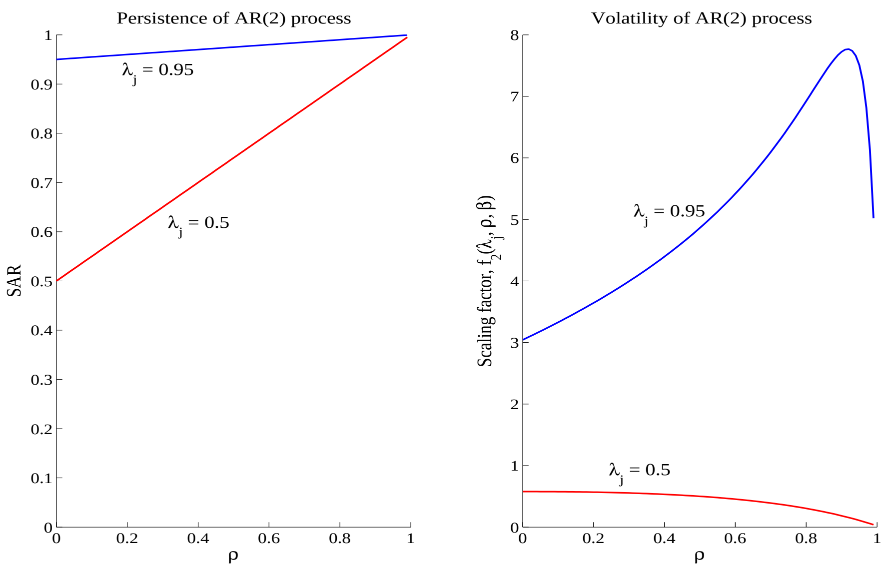
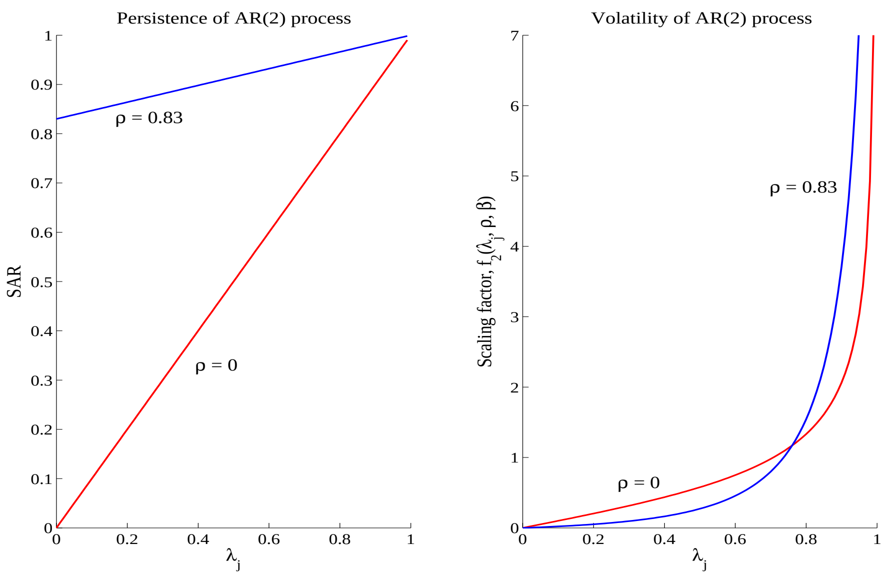
Figure 1: Persistence and volatility of Calvo model: function of money growth parameter(ρ)



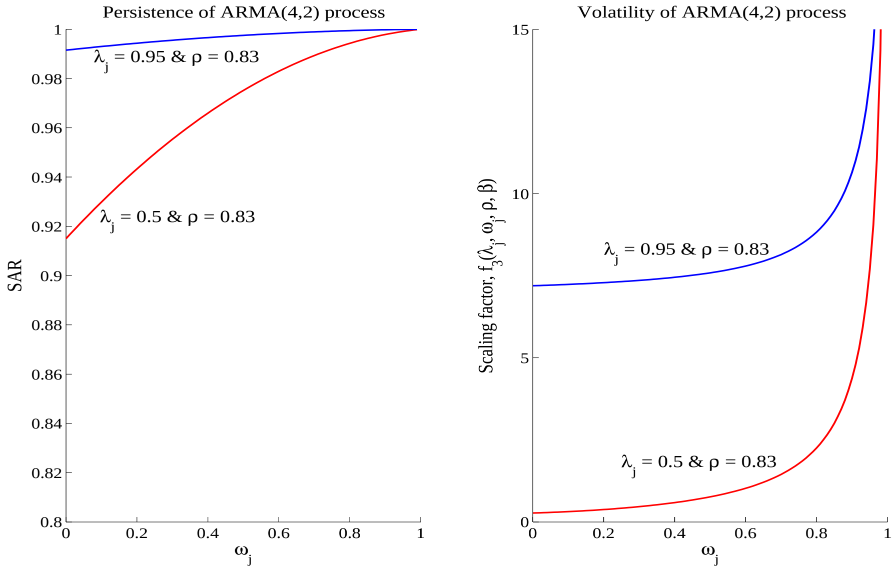
NOTES: The discount factor β is 0.99.

Figure 2: Persistence and volatility of Calvo model: function of Calvo parameter(λ_j)



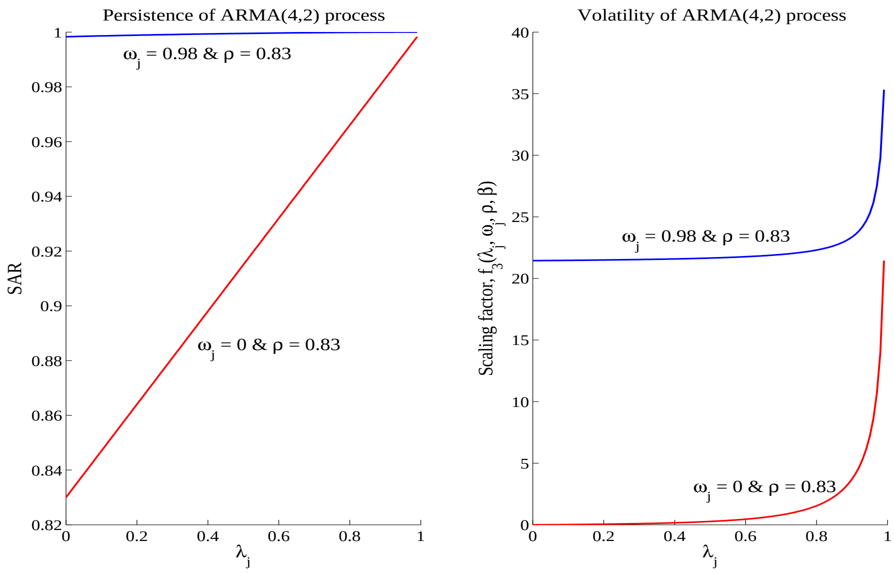
NOTES: The discount factor β is 0.99.

Figure 3: Persistence and volatility of dual stickiness model: function of information stickiness parameter(ω_j)



NOTES: The discount factor β is 0.99.

Figure 4: Persistence and volatility of dual stickiness model: function of Calvo parameter(λ_j)



NOTES: The discount factor β is 0.99.

Figure 5: The cross-border city pairs in the U.S. and Canada

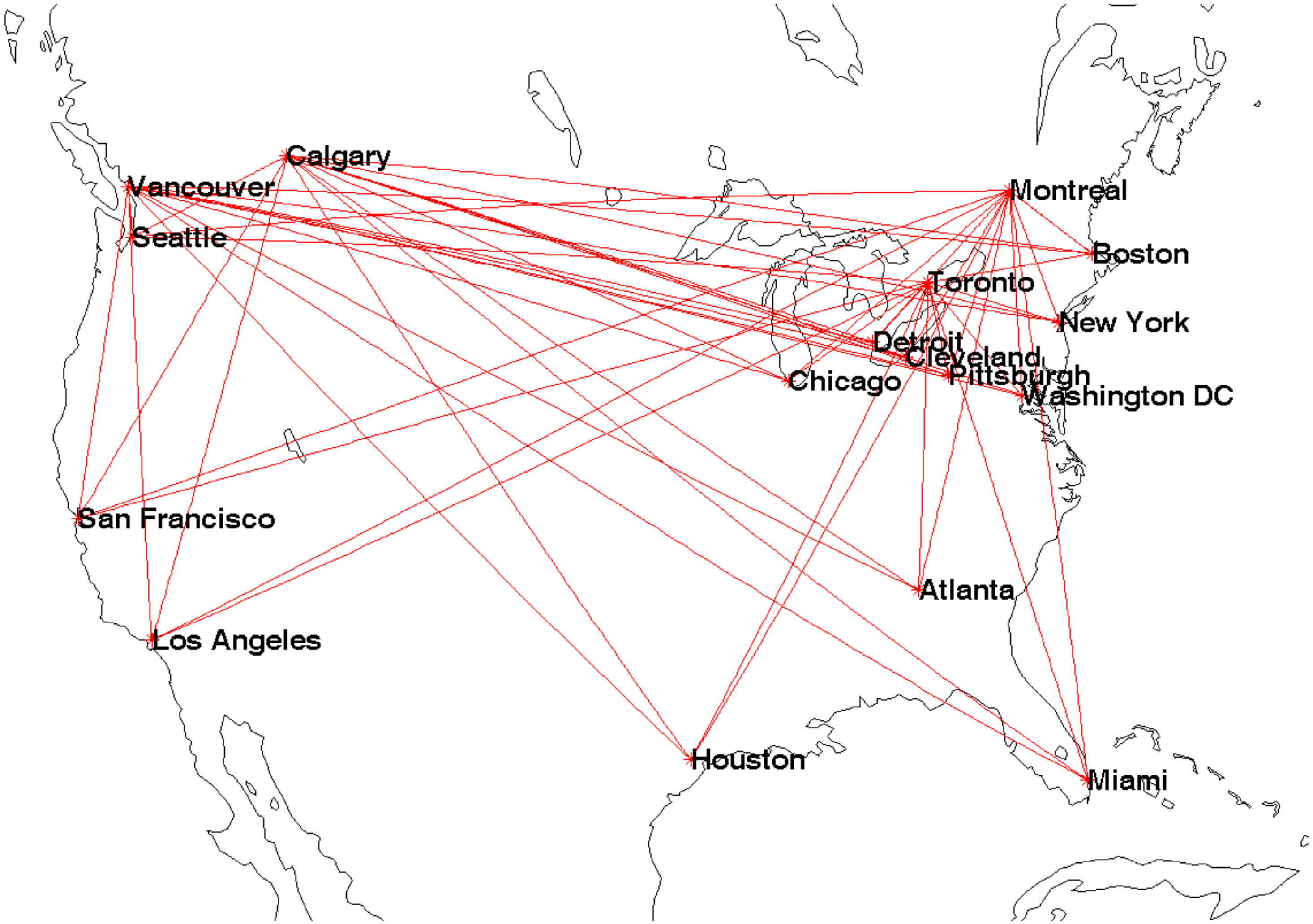


Figure 6: Real exchange rate persistence and price stickiness: Bils and Klenow (2004)

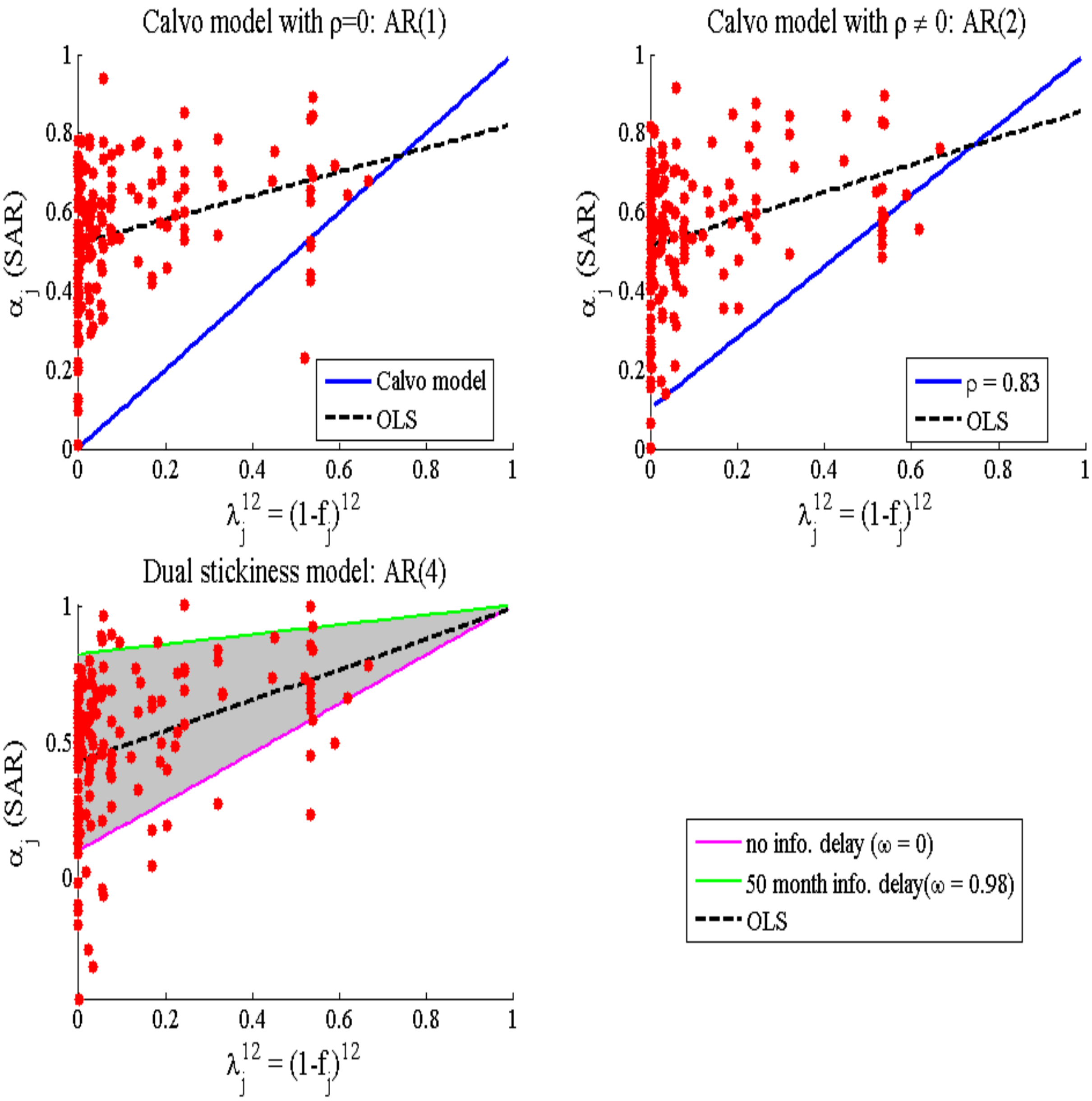


Figure 7: Real exchange rate persistence and price stickiness: Nakamura and Steinsson (2007)

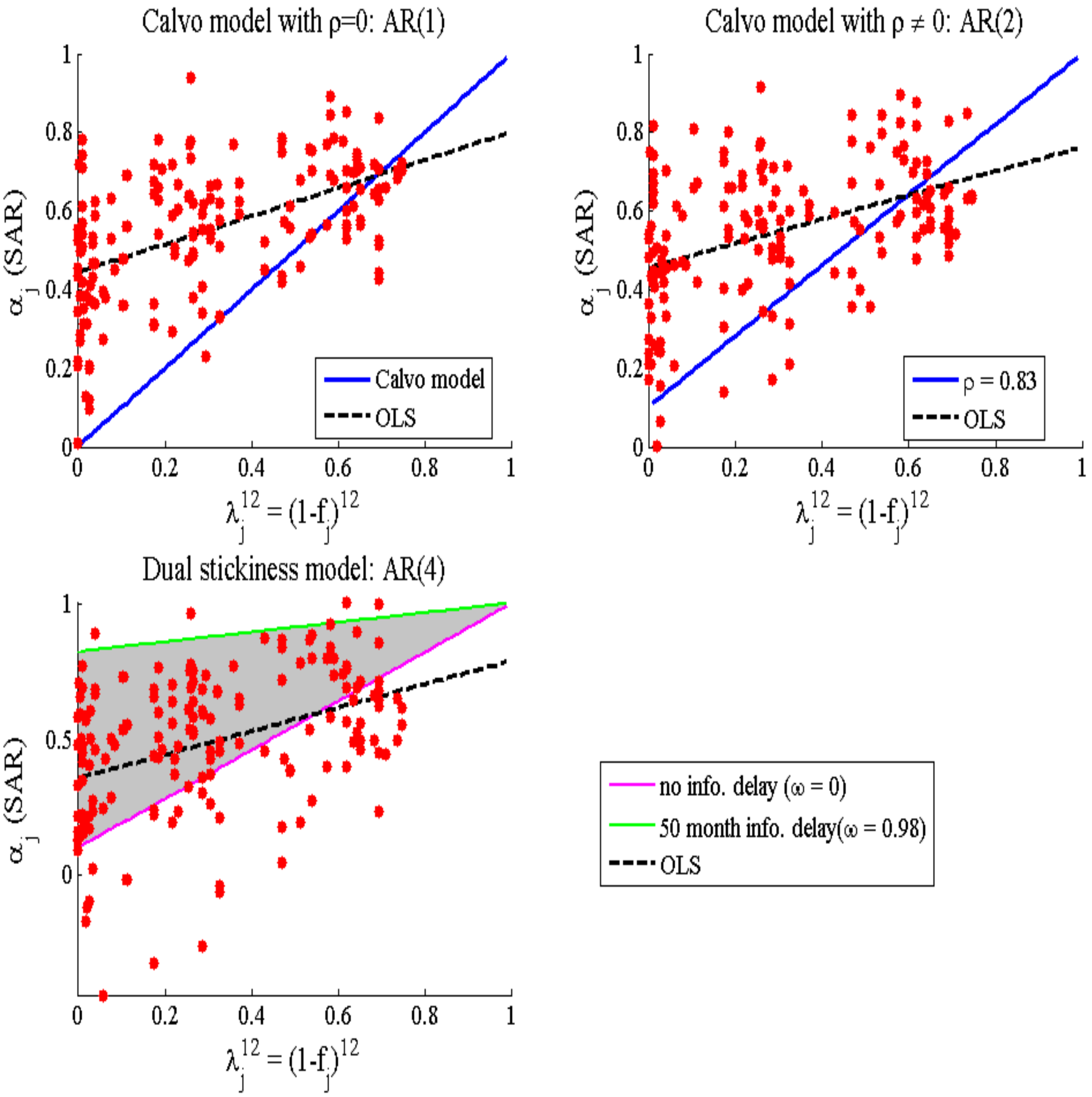
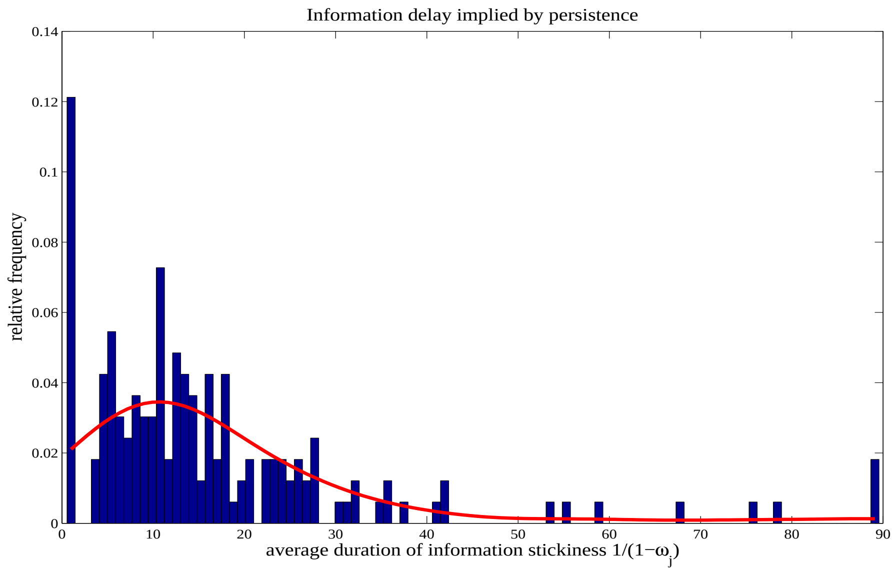
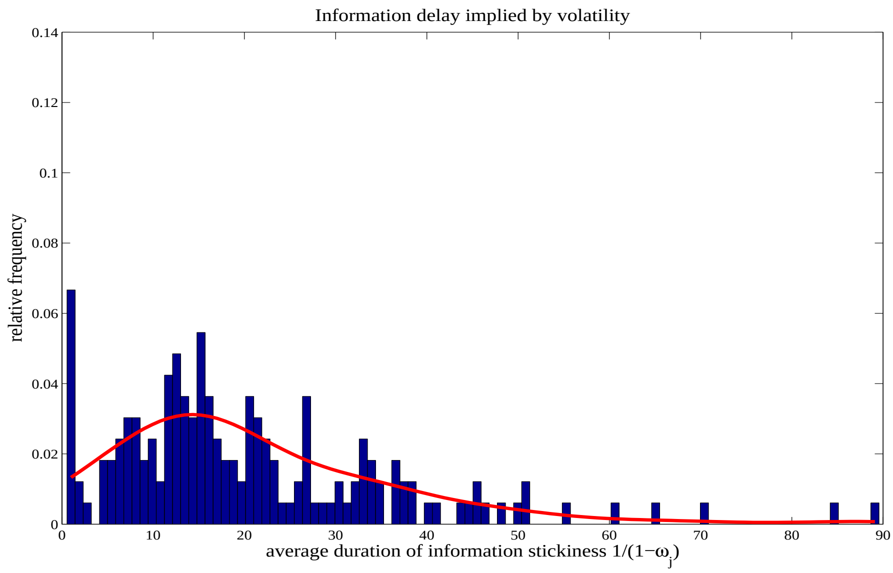


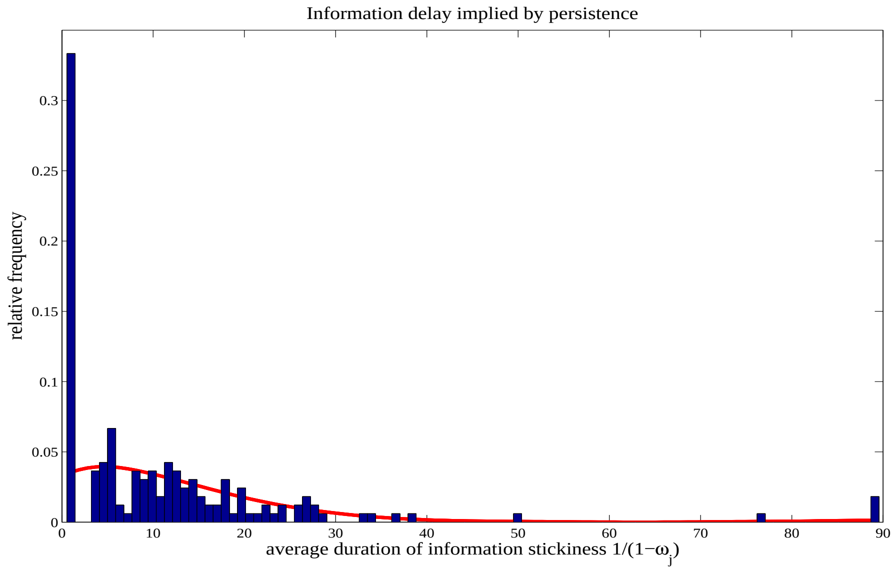
Figure 8: Empirical distribution of average information delay $1/(1-\omega_j)$: Bils and Klenow (2004)

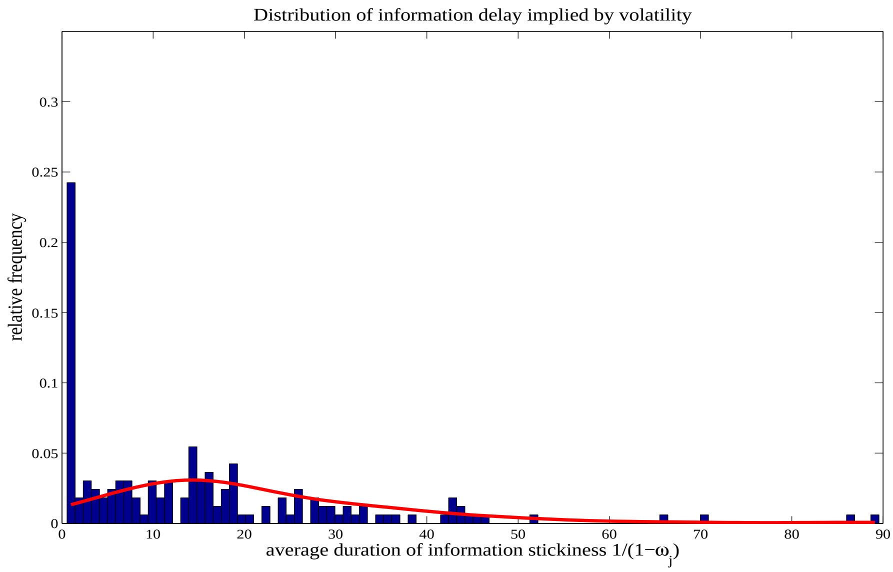




NOTES: The upper panel shows the histogram of average information delay $1/(1-\omega_j)$ where $\omega_j \in [0,1)$ for each good j is obtained by minimizing the distance between the observed SAR and theoretical prediction from the dual stickiness model using $\lambda_j = 1 - f_j$ from Bils and Klenow (2004) and $\rho = 0.83$. The smoothed lines are kernel density estimates. The lower panel shows the distribution when each ω_j is obtained by minimizing the distance between the observed volatility and theoretical prediction using $\lambda_j = 1 - f_j$, $\rho = 0.83$ and $\beta = 0.99$.

Figure 9: Empirical distribution of average information delay $1/(1-\omega_j)$: Nakamura and Steinsson (2007)





NOTES: See the notes of Fig.8. Nakamura and Steinsson's (2007) frequency of price changes, in stead of Bils and Klenow's (2004), is used for $\lambda_j = 1 - f_j$ in the computation of the theoretical value.

Table 1: Proportions of explained persistence of good-level real exchange rates: Bils and Klenow (2004)

The Calvo model with various ρ

ρ	0	0.3	0.5	0.7	0.9	0.95	0.98	0.946
Theory/Data	0.059	0.058	0.058	0.088	0.634	1.043	1.425	1.000

The dual stickiness model with $\rho = 0.83$

ω	0	0.3	0.5	0.7	0.9	0.95	0.98	0.930
Theory/Data	0.306	0.323	0.323	0.350	0.792	1.209	1.529	1.000

NOTES: The first panel shows the median ratio of the predicted persistence from the Calvo model to the observed persistence from data. Theoretical value of persistence is SAR based on $\lambda_j = 1 - f_j$ from Bils and Klenow (2004) for various ρ. The second panel shows the median persistence ratio when the theoretical value is computed from the dual stickiness model using $\rho = 0.83$ and various common ω. The last column of each panel shows the value of ρ and ω, respectively, giving the median ratio closest to one. The observed persistence is the SAR from the dynamic panel estimation. The median of the SAR estimates for AR(1), AR(2) and AR(4) models is 0.563, 0.568, and 0.508, respectively.

Table 2: Proportions of explained persistence of good-level real exchange rates: Nakamura and Steinsson (2007)

The Calvo model with various ρ

ρ	0	0.3	0.5	0.7	0.9	0.95	0.98	0.923
Theory/Data	0.484	0.505	0.506	0.549	0.922	1.226	1.522	1.000

The dual stickiness model with $\rho = 0.83$

ω	0	0.3	0.5	0.7	0.9	0.95	0.98	0.895
Theory/Data	0.664	0.659	0.660	0.681	1.015	1.329	1.619	1.000

NOTES: See the notes of Table 1. Nakamura and Steinsson's (2007) frequency of price changes, instead of Bils and Klenow's (2004), is used for $\lambda_j = 1 - f_j$ in the computation of the theoretical value.

Table 3: Proportions of explained volatility of good-level real exchange rates: Bils and Klenow (2004)

The Calvo model with various ρ

ρ	0	0.3	0.5	0.7	0.9	0.95	0.98	0.521
Theory/Data	0.130	0.143	0.153	0.148	0.096	0.064	0.036	0.153

The dual stickiness model with $\rho = 0.83$

ω	0	0.3	0.5	0.7	0.9	0.95	0.98	0.940
Theory/Data	0.125	0.149	0.181	0.276	0.691	1.129	1.950	1.000

NOTES: The first panel shows the median ratio of the predicted volatility from the Calvo model to the observed volatility from data. Theoretical volatility is the standard deviation of real exchange rates predicted by the observed standard deviation of nominal exchange rate changes combined with $\lambda_j = 1 - f_j$ from Bils and Klenow (2004) for various ρ. The second panel shows the median volatility ratio when the theoretical value is computed from the dual stickiness model using $\rho = 0.83$ and various common ω. The last column of each panel shows the value of ρ and ω, respectively, giving the median ratio closest to one. The observed volatility of real exchange rate is the extracted volatility component due to a time specific shocks in the two-way error component model.

Table 4: Proportions of explained volatility of good-level real exchange rates: Nakamura and Steinsson (2007)

The Calvo model with various ρ

ρ	0	0.3	0.5	0.7	0.9	0.95	0.99	0.801
Theory/Data	0.234	0.298	0.351	0.403	0.398	0.312	0.212	0.426

The dual stickiness model with $\rho = 0.83$

ω	0	0.3	0.5	0.7	0.9	0.95	0.98	0.916
Theory/Data	0.423	0.449	0.478	0.562	0.882	1.311	2.082	1.000

NOTES: See the notes of Table 3. Nakamura and Steinsson's (2007) frequency of price changes, instead of Bils and Klenow's (2004), is used for $\lambda_j = 1 - f_j$ in the computation of the theoretical value.

Table 5: Intervals between information update

		one month or less	1.01-5.99 months	6-11.99 months	12 months or above
Blinder et. al.'s (1998) survey data		25.6	13.2	16.5	44.5
Bils and Klenow	Persistence	11.5	8.5	26.7	53.3
	Volatility	6.1	4.2	18.2	71.5
Nakamura	Persistence	33.3	12.7	18.2	35.8
and Steinsson	Volatility	21.8	13.9	14.5	49.7

NOTES: The numbers in the first row represent the distribution, in percentages, of the frequency of price reviews reported in Blinder et al. (1998, Table 4.7 in p. 90). The second and third rows show the distribution of average information delay implied by the observed persistence and volatility of real exchange rates based on Bils and Klenow's (2004) data on the frequency of price changes. The fourth and fifth rows show the distribution of average information delay when Nakamura and Steinsson's (2007) data on the frequency of regular price changes is used.

Table A1: Frequency of price changes and information updates by category

ELI	Category name	Price Bils & Klenow	Information implied by Per.	Information implied by Vol.	Price Nakamura & Steinsson	Information implied by Per.	Information implied by Vol.	# of goods
FA	Cereals and cereals products	26.5	11.1	4.9	11.5	100.0	6.9	7
FB	Bakery products	25.7	5.5	4.4	9.8	8.7	6.8	1
FC	Beef and veal	47.2	12.2	4.8	25.5	13.8	5.5	8
FD	Pork	47.9	10.2	3.8	23.2	12.3	4.4	6
FF	Poultry	39.4	53.0	2.7	16.6	53.6	3.1	2
FG	Fish and seafood	42.4	8.7	10.6	20.4	9.7	15.2	1
FH	Eggs	61.8	7.5	6.5	47.6	7.5	6.8	1
FJ	Dairy and related products	33.7	6.7	4.4	24.9	7.2	5.3	4
FK	Fresh fruits	36.4	7.5	5.6	16.6	17.3	6.9	8
FL	Fresh vegetables	62.4	24.4	3.4	40.8	25.6	3.6	6
FM	Processed fruits and vegetables	24.9	5.2	4.1	10.5	7.7	6.0	6
FN	Juices and nonalcoholic drinks	35.6	6.1	2.4	13.1	8.2	2.9	4
FP	Beverage incl. coffee and tea	21.1	8.8	7.3	8.9	18.1	13.2	11
FR	Sugar and sweets	22.9	4.8	7.0	9.9	7.1	12.7	2
FS	Fats and oils	29.5	14.5	6.1	18.1	16.0	6.7	8
FV	Food away from home	9.0	3.6	12.9	5.0	5.9	88.5	3
FW	Alcoholic beverages at home	19.3	6.1	6.8	10.6	7.5	10.0	7
FX	Alcoholic beverages away from home	6.4	2.4	14.1	5.0	3.0	25.1	1
HB	Lodging away from home	38.1	11.2	4.9	41.7	11.2	4.8	2
HF	Gas and electricity	43.4	3.6	5.3	38.1	3.6	5.4	1
HK	Appliances	19.0	2.7	3.6	3.6	15.0	25.6	2
HL	Other equipment and furnishings	16.1	10.2	6.6	2.8	100.0	100.0	1
HN	Housekeeping supplies	19.2	9.1	3.2	9.4	60.0	5.7	8
HP	Household operations	6.5	6.7	38.6	4.3	10.8	100.0	1
AA	Men's apparel	26.0	3.1	7.5	4.5	11.3	100.0	5
AB	Boy's apparel	25.9	2.4	11.5	4.3	6.9	100.0	1
AC	Women's apparel	45.0	6.3	6.8	2.5	100.0	100.0	6
AE	Footwear	28.0	4.8	7.1	3.5	60.0	100.0	2
AF	Infants' and toddlers' apparel	36.3	7.6	7.8	3.5	100.0	100.0	2
TA	New and used motor vehicles	39.1	7.5	5.7	31.3	7.6	6.0	7
TB	Motor fuel	78.9	11.3	6.3	88.6	11.3	6.2	1
TD	Motor vehicle maintenance and repair	11.6	6.7	6.1	10.7	7.1	6.4	2
TE	Motor vehicle insurance	15.5	3.2	11.8	8.2	4.6	27.7	1
TG	Public transportation	5.0	4.3	19.8	4.4	4.9	31.2	3
MB	Nonprescription drugs and medical supplies	13.7	5.8	14.8	7.9	8.7	42.6	2
RA	Video and audio	22.0	10.3	10.2	9.4	55.7	24.8	2
RD	Photography	8.6	9.6	16.2	8.8	12.0	30.5	2
RF	Recreation services	8.8	6.7	13.3	9.0	6.6	12.9	1
RG	Recreational reading materials	12.4	15.1	34.5	5.4	100.0	100.0	3
GA	Tobacco and smoking products	21.6	4.3	1.3	23.2	4.3	1.3	4
GB	Personal care products	11.1	4.7	10.8	3.9	14.7	100.0	10
GC	Personal care services	4.1	78.7	100.0	3.1	100.0	100.0	2
GD	Miscellaneous personal services	5.1	13.8	100.0	3.0	100.0	100.0	8

NOTES: ELI in the first column stands for the entry level item in the CPI. EIU price series for good and service used in the analysis are matched to BLS's ELI codes. The third column shows the median value of average monthly frequencies of price changes from Bils and Klenow (2004), among the goods included in each category. The fourth and fifth columns show the median value of the estimated average monthly frequencies of information updates implied by the persistence (Per.) and volatility (Vol.) of good-level real exchange rates, when Bils and Klenow (2004) is used to compute the theoretical prediction. The sixth column is the median of the frequencies of regular price changes from Nakamura and Steinsson (2007). The seventh and eighth columns show the median of frequencies of information updates when Nakamura and Steinsson's (2007) data is used. The last column shows the total numbers of goods and services included in each category of ELI codes.