

**Dynamic Efficiency and Asset Bubbles**  
**under Financial Market Friction <sup>#</sup>**

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**Abstract**

This paper investigates whether the criterion for the real-interest-rate versus the-economic-growth-rate is effective in evaluating dynamic efficiency/inefficiency and understanding rational bubbles in a dynamic economy with frictions of financial markets. In the closed economy, this criterion is effective for the sustainable bubbles, but not in terms of dynamic efficiency/inefficiency. Asset bubbles do not necessarily restore efficiency, but rather asset bubbles typically coexist with capital under-accumulation. The effectiveness of that criterion is restored in the small-open economy if the output produced by borrowers is international collateral.

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## 1. Introduction

The criterion for the real interest rate versus the economic growth rate is a problem of theoretical and empirical importance in evaluating the efficiency of the economy and understanding the sustainability of asset bubbles. Diamond (1965), Ihori (1978), and Tirole (1985) demonstrate that when the return to capital is equal to the real interest rate, this criterion becomes a benchmark for understanding dynamic efficiency and sustainability of bubbles. However, various kinds of financial frictions, including uncertainty, transaction costs, asymmetric information, and other incentive problems will deter this equality to continue to hold in the actual economy so that the effectiveness of this criterion is an open question. Abel et al (1989) and Bohn (1995) provide examples in which the safe interest rate is smaller than the economic growth rate due to risk premium but the economy is dynamically efficient.

This problem is stringent also from the empirical prospect because the current speculative bubbles will be associated with the historically low real interest rate. Figure 1 illustrates average interest rates and the average economic growth rate of G7 countries (U.S., U.K., Japan, Germany, France, Canada, and Italy).<sup>1</sup> Remarkably, before the end of 1990s, all the interest rates were greater than the economic growth rate except for the deposit rate, but after that, many of the interest rates began to be smaller than the economic growth rates. Since 2000 and around, several kinds of bubbles recurrently emerged, including IT bubbles, housing market booming in US, the global booming in the stock markets, and appreciations in gold and oil prices.<sup>2</sup>

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<sup>1</sup> Each of interest rates and the economic growth rate is a simple average of G-7 countries (*source: IFS*). We exclude the 1991 data of German in calculating averages. Money market rate, treasury bills, treasury bills: 3years or longer, deposit rate, and max overdraft reflect call rate, short-term rate of the government bond, long-term rate of the government bond, short-term deposit interest rate, and loan interest, respectively.

<sup>2</sup> Additionally, behind asset bubbles in China that have been sustained for more than two decades is the far higher economic growth rate than the interest rates.

Tirole (1985) demonstrates that asset bubbles arise when capital is overly accumulated in the bubbleless economy. His finding is, however, inconsistent with the current world economy in that Tirole (1985) can not explain the coexistence of “global bubbles” and weak incentive for investment relative to great global savings, in other words, the “asset shortage” following the terminology of Caballero (2006). Caballero and Krishnamurthy (2006) develop an insightful model of a small-open economy in which capital under-accumulation and asset bubbles can coexist. In theirs, asset bubbles are used for collateral for financing productive investment. Their contribution is to show the complementary role of bubbles with capital accumulation, in contrast with Tirole where bubbles and capital accumulation are substitutable.

The purpose of this paper is to investigate the effectiveness of the criterion for the real-interest-rate versus the-economic-growth-rate in evaluating dynamic efficiency/inefficiency and understanding rational bubbles in the presence of financial market frictions. In doing so, we construct a simple model which is as close as possible to Diamond (1965), Ihori (1978), and Tirole (1985) except for that there is a friction in the financial market. As the extensive literature argues, borrowing constraints or credit rationing that arise in response to the friction of financial markets leads to the breakdown of the equality between the return to capital and the real interest rate (e.g., Stiglitz and Weiss (1981), Gale and Hellwig (1985), Williamson (1986), Bernanke and Gertler (1989), and others). The discrepancy between the two rates has different results on dynamic efficiency and sustainability of asset bubbles, and the criterion relative to the one in the standard economy.

In the closed economy, the criterion for the real interest rate versus economic growth rate is effective for the sustainable bubbles, but not in terms of dynamic efficiency/inefficiency. Asset bubbles move the capital stock down to the smaller level than the Golden Rule so that asset bubbles and capital under-accumulation coexist. Asset bubbles do not in general restore

dynamic efficiency, but do only if capital is too overly accumulated in the bubbleless economy.

The theoretical finding explains asset bubbles, asset shortage, and the low real interest rate simultaneously, all of which features the current world economy. The effectiveness of that criterion is restored in the small-open economy with the real interest rate being constant if the capital good becomes international collateral. In the small-open economy, asset bubbles never crowd out capital accumulation, but has the role of raising the real interest rate faced by domestic investors.

This paper is organized as follows. Section 2 sets up the model and studies the benchmark economy. Section 3 analyzes the closed economy when there is the friction of financial market, and Section 4 the small-open economy. Section 5 concludes.

## 2. Basic Model

Let us consider an economy of overlapping generations that lasts for infinity. At each period  $t = 0, 1, 2, \dots, \infty$ , the economy is populated by a continuum of ex ante identical agents that live for two periods. Letting  $N_t$  denote the number of young people at  $t$ , the population grows at rate  $n > 0$ , satisfying  $N_t = (1+n)^t N_0 = (1+n)^t$ . At each period the final good is produced by firms that use labor and capital as inputs according to the constant-returns-to-scale technology described as  $Y_t = F(K_t, N_t)$ , where  $K_t$  and  $N_t$  are aggregate supplies of capital and labor, and  $Y_t$  is the output of the final good. That technology is described as a per-capita form by  $y_t \equiv Y_t/N_t = F(K_t/N_t, 1) \equiv f(k_t)$ , where  $k_t$  is the capital-labor ratio and  $y_t$  is the per-capita output of the final good.  $f(\cdot)$  is thrice continuously differentiable, increasing, concave, satisfying  $f(0) = 0$ , and  $\lim_{k_t \rightarrow 0} f'(k_t) = +\infty$ . Since the production technology is homogeneous of degree one, output of the final good can be described in terms of the action of a