A Global Analysis of Financial Market Integration [?]

Masaya Sakuragawa and Hiroto Takahashi[#]

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Preliminary, Comment Welcome

Abstract

We consider a global analysis of financial market integration by modeling a two-country overlapping generations economy in the presence of financial market frictions, and find several important features that have not been obtained from the local analysis around steady states. There are a stable symmetric steady state with harmonized growth and stable asymmetric steady states with capital flows from the poor to the rich, both of which may coexist, and out of steady states, there are interesting non-monotone behavior of the pattern of capital movement and development. The existence of a stable asymmetric steady state does not necessarily recommend the poor to keep capital control indefinitely, but if the stable symmetric steady state coexists, will guide the poor to open eventually capital accounts by delaying the timing of liberalization until arriving at some development stage. The existence of the unique steady state that is stable and symmetric may or may not recommend the poor to lift capital controls promptly because the symmetric-breaking process will occur in the transition. The concept of optimal timing for liberalization allows us to explore conditions for successful integration. Conditions for successful integration depend on several characteristics of the levels of per-capita- income, the distribution, enforcement technology, global savings, and TFP.

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[#] Keio University, Japan, masaya@econ.keio.ac.jp, hirotot@gs.econ.keio.ac.jp

1. Introduction

Globalization, which is meant by enhanced trade and financial integration, should promise greater prosperities through the channels of improved allocation of resources in the world. How global is the world economy in reality? Contrary to the conventional wisdom from the neoclassical economic theory, international integration remains limited, remarkably so in financial markets. Since the influential paper by Feldstein and Horioka (1981), much literature has addressed that global financial markets are far from complete.

Facing incomplete financial markets, how the world economy reaps the efficiency benefits of financial integration is a subject of great importance. Indeed, observers and policymakers of developing countries are skeptical on the effects of capital account liberalization on economic growth, but do not appear to intend to isolate their home country "indefinitely" from the world financial market.

The notion of timing for financial liberalization provides a hint for thinking how financial market integration is progressed. McKinnon (1991) addressed this question in terms of the sequence of liberalization between trade and capital markets.[2] [3] As a matter of fact, Japan, Korea, and Taiwan are successful evidences. All of these countries attained miraculous growth, but did not liberalize their capital markets until 1980s. The notion of the optimal timing for labialization raises several questions. Whether late-developing countries should open capital accounts? Is there an optimal timing for opening capital accounts so as to attain successful development? From the prospect of the world economy, what determines the period for successful financial integration? In this paper we address these questions from a theoretical perspective.

In this paper, we consider a two-country overlapping generations model in the presence of financial market frictions. Two countries are assumed to be inherently identical in production

[2] Bartolini and Drazen (1997) emphasize the signaling role of capital account liberalization as a commitment to policy reforms to boost capital inflows.

[3] Braun and Raddatz (2007) report that trade liberalization occurred before capital account liberalization in 68 of the 73 countries that liberalized any of these dimensions between 1970 and 2000.

technologies and institutions for contract enforcement, but differ only in their initial levels of income.

The model is standard except for one respect; entrepreneurs have no initial wealth for financing productive investment but can pledge up the future income (wage income in the model) as collateral. This assumption is motivated to make the dynamical system a one-dimensional map of the capital-labor ratio of either country and to avoid a complicated dynamical system (e.g., Boyd and Smith (1997) and Matsuyama (2004)). This assumption also enables us to preserve the non-monotone behavior of the interest rate in terms of the capital-labor ratio that is the source of multiple steady states and implicitly involved in the preceding literature. In order to reach our goal, the theoretical research is needed to provide characterizations on the short-run and the long-run inequality of income between countries, the patterns of capital movement, the timing of financial liberalization, and conditions for successful integration.

We find a number of global properties that are not found in the steady state analysis. There are a stable symmetric steady state with equality in per-capita income and stable asymmetric steady states with an inequality in per-capita income followed by capital flows from the poor to the rich, and both of which may coexist. Out of steady states, there are complicated but interesting non-monotone behavior of the pattern of capital movement and development. We show that how the process of development goes hand in hand with financial development, how the pattern of capital movement and development evolves over time, and how and when the globalization magnifies and lessens the divergence in per-capita income between the rich and the poor.

From the prospect of late-developing countries, the timing of liberalizing the home financial market is a stringent issue for successful development. The existence of a stable asymmetric steady state does not necessarily recommend the poor to isolate their countries indefinitely from the world financial market. If the stable symmetric steady state also exists, the poor can go on the successful path by delaying the timing of liberalization until arriving at some development stage. On the other hand, the existence of the unique steady state that is stable and symmetric does not guarantee late-developing countries to attain the successful development by lifting capital controls quickly. The world financial market involves the reversal of the direction of capital flows, and the poor are trapped into stagnation with capital flight in early development stage.

The notion of optimal timing for liberalization allows us to explore conditions for successful integration in which all countries attain the same capital-labor ratio. From the prospect of financial integration, conditions for successful integration depend on several characteristics of the levels of per-capita income, its cross-country distribution, enforcement technology, global savings, and TFP. Higher levels and more equality in per-capita income are determinants of successful integration. Furthermore, the greater TFP level, the development of contract enforcement, or the greater global savings shortens the waiting time for successful financial integration.

This paper is organized as follows. Section 2 reviews the related literature. Section 3 sets up the model. Section 4 characterizes the autarkic economy and section 5 the world economy. Section 6 analyzes the dynamic properties and provides policy implications for financial integration.

2. Related Literature

A number of approaches have attempted to answer the question why capital does not flow from the rich to the poor, and sometimes even flow from the poor to the rich. Among several research lines, one research line emphasizes the role of frictions in international financial markets, and recently focuses on legal and institutional infrastructure and asymmetric information problems as obstacles to smooth capital flows (e.g., La Porta et al (1997, 1998), Levine (1998), Levine et al (2000), and others).[4]

[4] Another research line has emphasized the cost of the governments' time inconsistency problem associated with sovereign debt. The literature includes Bulow and Rogoff (1989), Eaton and Fernandes (1995), Tirole (2003), and Broner and Ventura (2007). The two research lines are complementary in terms of enforceability. The corporate finance approach and the sovereign debt approach may be classified according to whether enforceability is exogenous or endogenous.
On the other hand, the research line supporting the "Lucas Paradox "(1990) has a negative stance to the financial-friction view, arguing that poor countries also have lower endowments of factors complementary with physical capital, and hence the large difference in capital-labor ratio will

The empirical literature includes Bekaert et al (2001), Reinhart and Rogoff (2004), Portes and Rey (2005), Chin and Ito (2006), Alfaro et al (2006), and Braun and Raddatz (2007). Much theoretical literature has explained why and how perverse capital movements and the divergence in income between the rich and the poor arise under financial market integration. The theoretical literature includes Gertler and Rogoff (1990), Boyd and Smith (1997), Sakuragawa and Hamada (2001), and Matsuyama (2004), Caballerro and Krishnamurty (2001) (2006), Aghion et al (2004), and Aoki, Benigno and Kiyotaki (2006), and others.[5]

Gertler and Rogoff (1990) is a pioneering theoretical model that explained the perverse capital movement using the corporate finance approach. In their static model, the difference in borrowers' ability to rely on external finance causes capital flows to go from poor to rich. Boyd and Smith (1997) developed this idea, demonstrating that financial market integration causes the divergence in income between the poor and the rich, even with identical technologies and institutions for contract enforcement. Matsuyama (2004) constructed more general but tractable model than Boyd and Smith (1997), characterizing all the steady states, including those with binding borrowing constraint and the one without it. The analysis of Boyd and Smith (1997) and Matsuyama (2004) depends on the poverty trap argument that involves the multiplicity of steady states that arises from a feedback effect between the borrowers' wealth and the aggregate investment. But the introduction of the feedback effect makes the analysis so complicated that their scope turned out to be limited to the local analysis around steady states. Sakuragawa and Hamada (2001) are an exception that analyzed the effects of lifting capital controls on economic development, in and out of steady states. They demonstrated that late-developing countries may have an optimal timing for successful development in a model of two countries with different institutions for contract enforcement, and with identical technological externalities in production. We comment on other related works that link contract enforcement with international integration. Tornell and Velasco (1992) model the lack of contract enforcement as a discrepancy

coexist with the equalization of marginal product of capital. Along this view, Caselli and Feyrer (2007) find that the cross-country difference in marginal products of capital is not so large relative to the difference in capital-labor ratio, and support partially Lucas's argument.

[5] Caballerro and Krishnamurty (2001) (2006) and Aghion et al (2004), investigate the small-open-economy model of asymmetric information in order to identify perverse capital flows that happened in Asian emerging market countries.

between social and private rate of return to capital that rises from poorly established property rights, demonstrating that opening capital accounts, despite the high social return, leads to the slowdown of economic growth and capital flight. Greif (1994) offers interesting contrasting studies of two groups with different internal enforcement systems, arguing that the rule-based governance has an advantage to globalization over the relation-based governance. Acemoglu and Zilibotti (1997) model the lack of enforcement as incompleteness risk diversification, demonstrating that the market incompleteness, combined with technological non-convexities, magnifies the inequality of nations and becomes a reason for perverse capital mobility. Dixit (2003) analyzes the effect of contract enforcement on trade expansion, demon straing that in the small trading world human-based governance is dominant, but as the world is greater, has to be replaced by the rule –based governance. Broner and Ventura (2007) model globalization as a gradual improvement in technology that increases the fraction of tradable goods, demonstrating that when countries can not commit to pay their debts, but can control enforceability, globalization might lower domestic asset trade, worsening risk sharing and lowering welfare. Tirole (2003) is an excellent works that demonstrates that enforceability to foreign debts affects crucially the quality of the home financial markets.

3. The Model

The model is based on an overlapping generation model that consists of two period lived agents. Time is discrete, and during each period $t = 0, 1, \ldots$ a set of agents is born. Each generation consists of a continuum of agents of unit mass. There is no population growth.

A single final good is produced using capital stock and labor as inputs. Let K_t denote the capital stock in period t, and L_t denote the labor supply in period t. Then, the output of the final good in period t, Y_t, is produced according to the production function given by $Y_t = A_t F(K_t, L_t)$, where $F(.)$ exhibits the property of is a constant returns to scale production function and A_t is the TFP at period t. We set $A_t = 1$ so long as unnecessary. We denote $k_t \equiv K_t / L_t$ and $f(k_t) \equiv F(k_t, 1)$, and assume that $f(0) = 0$, $f'(k) > 0 > f''(k)$ and $f'(0) = \infty$. It is also assumed that the factor markets are competitive. The firm's profit maximizing leads to $r_t = f'(k_t)$ and $w_t = f(k_t) - k_t f'(k_t)$, where r_t is the rental rate on capital and w_t is the real wage rate. Let $W(k_t) \equiv f(k_t) - k_t f'(k_t)$, then $f''(k) < 0$ implies that $W'(k_t) > 0$ holds for all $k > 0$, and in addition we assume that

(A1) $\quad W'(0) = \infty, \quad W''(k) < 0.$

For simplicity, capital is assumed to depreciate fully in one period.

Within each generation, agents are divided into two types, "investors" and "entrepreneurs". A fraction a $(0 < a < 1)$ of agents are investors, each of whom is endowed with one unit of labor in the young age. The remaining fraction $1 - a$ of agents are entrepreneurs, each of whom is *not* endowed with labor in the young age, but has access to a single indivisible investment project for converting the final good to the capital good after one period; one unit of the final good invested in this project in period t yields R units of capital in period $t+1$. In addition, each of them is endowed with one unit of labor *in the old age*. After having completed the investment project and sold the produced capital good to the final-good firm, each of them supplies labor to the final-good firm. Both types of agents are risk neutral, and care only about old-age consumption. In addition, investment projects are not transferable among entrepreneurs so that any one of them runs at most one project.

Investors receive wages by supplying one unit of labor elastically in the labor market in the young age, and lend all the earned income to others with a safe (gross) interest rate r_{t+1} which is exogenously given to agents but endogenously determined in the model. Entrepreneurs will start the investment project by borrowing or do nothing when young.

We assume that the legal enforcement is imperfect so that it is difficult to fulfill financial contracts between investors and entrepreneurs. Entrepreneurs can repudiate the obligation by hiding a fraction $1 - l$ $(0 < l < 1)$ of the revenue generated from the investment project so that, in case of default, investors can seize only a fraction l of the produced capital good. The parameter l captures a level of legal infrastructure for contract enforcement. Breaching the contract yields another cost to entrepreneurs. If entrepreneurs breach the contract, they are sent to prison, and lose the opportunity of earning wage incomes by supplying labor to the final-good firm. Alternatively, they may be discriminated for job opportunities for the reason of criminal record. The latter will reflect a kind of social punishment that will be made out of court, the cost of which increases as the economy is rich.[6] This assumption is an artifact to motivate the observed fact that the cost of breaching contracts is higher in richer countries in which people have more chances of earning more money.

We are now ready to look at the investment decision. If potential entrepreneurs start an investment project by financing one unit of the final good in the financial market with a (gross) interest rate r_{t+1}, their old-age consumption is equal to $r_{t+1}R + w_{t+1} - r_{t+1}$, while otherwise, their old-age consumption becomes w_{t+1}. Potential entrepreneurs are willing to produce capital if

(1) $$Rf'(k_{t+1}) \geq r_{t+1}.$$

We refer to this inequality as *the profitability constraint*. However, even if (1) is satisfied, the enforcement problem may prevent entrepreneurs from financing their own projects. If entrepreneurs repay obligations honestly, they earn $Rr_{t+1} - r_{t+1} + w_{t+1}$, while if they breach the contract, they would obtain $(1-l)Rr_{t+1}$. Anticipating the possibility of the entrepreneur's strategic default, the investor supplies the fund only if the incentive compatibility is satisfied so that the entrepreneur can start the project only if

(2) $$lRf'(k_{t+1}) + W(k_{t+1}) \geq r_{t+1}.$$

This inequality implies that $(lRr_{t+1} + w_{t+1})/r_{t+1}$ should exceed unity, the amount of the requirement for starting the project. We shall thus call (2) *the borrowing constraint*. A greater value in l implies the LHS of (2) to be greater, and thus weakens the incentive to default and makes the borrowing constraint less binding. In other words, a high (low) l reflects the high (low) level of legal protection for creditors or shareholders, and will be associated with well (poorly) developed financial markets [e.g., La Porta et al (1997, 1998), Levine (1998), Levine et al (2000)].[7] The parameter l is interpreted to capture the efficiency of the broadly defined financial system, including the banking system, and stock and bond markets.

One important feature of our model is that entrepreneurs receive the wage income not in the

[6] Human-based enforcement is typical in societies with underdevepled legal enforcement. See for example, Townsend(1994), Udry(1994), and Banerjee andf Newman (1998)

[7] La Porta et al (1997) find that countries with poor investor protections, measured by both the character of legal rules and the quality of law enforcement, have smaller capital markets. Levine (1998) shows that financial depth is closely linked with measures of legal treatment of outside creditors developed by La Porta et al.

youth but in the old age. This assumption is an artifact to motivate entrepreneurs' incentive compatibility to be affected by the level of their wealth that is an increasing function of the capital-labor ratio. Following the standard overlapping generations model *a la* Diamond (1965), one might be motivated to assume that entrepreneurs use the wage income earned in the young age as internal wealth for financing investment (e.g., Bernanke and Gertler (1989)), but we have a different way. If the latter assumption is adopted, the dynamic analysis of the world economy with financial market integration will be far complicated. Boyd and Smith (1997) and Matsuyama (2004) had to confine their study on the local analysis around steady states. As analyzed in details below, our modification of the model enables us to conduct the global analysis in the world economy while preserving general features that arise from the financially constrained economy.

4. The Autarky Case

In this section, before considering the financial market integration between countries, we analyze the general equilibrium of the case of autarky. In the case of autarky, domestic investment is adjusted to be equal to domestic saving in equilibrium. The measure of domestic saving is $aW(k_t)$, while the measure of maximum domestic investment is $1-a$. Since each investment supplies R units of physical capital, the maximum physical capital is $(1-a)R$. We impose the following assumption.

(A2) $aW((1-a)R) < 1-a$.

The LHS of (A2) is the maximum allowable domestic saving, and hence (A2) implies that domestic saving is always less than the demand for funds by entrepreneurs. Under (A2), the demand for funds exceeds the supply of funds. Accordingly, domestic investment is equal to $aW(k_t)$. Since the aggregate amount of domestic investment is $aW(k_t)$ and each investment supplies R units of capital, the aggregate capital stock at period $t+1$ is described, in per-capita term, as

(3) $k_{t+1} = RaW(k_t).$

Equation (3) gives the equilibrium law of motion for capital stock in the case of autarky. (A1) ensures that (3) has a unique nontrivial steady state capital stock k^*, defined by $k^* = RaW(k^*)$. For any $k_0 > 0$, k_t converges monotonically to k^*. The equilibrium law of motion is illustrated in Figure 3-1.

The parameter l does not affect the dynamics of capital, but will affect the equilibrium interest rate. The two constraints, (1) and (2), can be summarized as

(4) $$r_{t+1} \le \text{Min } \{Rf'(k_{t+1}),\ lRf'(k_{t+1}) + W(k_{t+1})\}.$$

The market clearing in the financial market requires the inequality in (4) to be binding with equality, and is rewritten as

(5) $$r_{t+1} \equiv r(k_{t+1}) = \begin{cases} lRf'(k_{t+1}) + W(k_{t+1}) & if \quad k_{t+1} < K(l), \\ Rf'(k_{t+1}) & if \quad k_{t+1} \ge K(l), \end{cases}$$

where $K(l)$ is the threshold level of capital defined implicitly by $lRf'(K(l)) + W(K(l)) = Rf'(K(l))$.[8] The borrowing constraint is binding if $k_{t+1} < K(l)$, while the profitability constraint is binding if $k_{t+1} \ge K(l)$.

The fraction of entrepreneurs who are able to fund the project is equal to $aW(k_t)/(1-a)$, while the rest, $1-(aW(k_t)/(1-a))$, are not. If $k_{t+1} \ge K(l)$, entrepreneurs are indifferent between borrowing and not borrowing. However, if $k_{t+1} < K(l)$, they strictly prefer investment by borrowing. Thus the remaining $1-aW(k_t)/(1-a)$ of entrepreneurs are denied to receive the fund and hence the equilibrium allocation turns out to involve *credit rationing*.

The function $Rf'(k_{t+1})$ is monotonically decreasing, while $lRf'(k_{t+1}) + W(k_{t+1})$ is decreasing in k_{t+1} if $k_{t+1} < lR$ and increasing if $k_{t+1} > lR$. The two curves intersect at $k_{t+1} = K(l)$. Note that $K(l)$ is decreasing in l, with $K(0) > 0$ and $K(1) = 0$. There exists a $\hat{l} \in (0,1)$ such that $K(l) > lR$ if and only if l is less than $\hat{l}$. In Figure 3-2 $r(k_{t+1})$ is decreasing for all $k_{t+1} > 0$. This case occurs when l is high, satisfying $l \ge \hat{l}$. On the other hand, in Figure 3-3 $r(k_{t+1})$ is first decreasing for $k_{t+1} < lR$, later increasing for $lR < k_{t+1} < K(l)$, and finally decreasing for $k_{t+1} > K(l)$. The non-monotonicity arises

[8] If the inequality does not bind, domestic investment should be equal to $1-a$.

when l is small, satisfying $l < \hat{l}$.

Two conflicting effects give rise to the non-monotone feature; the effect of decreasing returns to capital and the effect of mitigating the borrowing constraint. As the level of wealth that borrowers pledge as collateral increases, the borrowing constraint becomes less stringent. An increase in borrowers' wealth mitigates the borrowing constraint, allowing them to demand more funds, and raising the interest rate up. At the low level of capital stock, the effect of decreasing returns to capital is dominant and thus $r(k_{t+1})$ is decreasing. As the level of capital increases beyond lR, the effect of mitigating the borrowing constraint begins to be dominant, and thus $r(k_{t+1})$ is increasing. Finally, as the level of capital increases beyond $K(l)$, the borrowing constraint ceases to be binding and thus the agency cost disappears; the effect of decreasing returns to capital makes $r(k_{t+1})$ decreasing.

The non-monotone feature is more likely to arise if l is small or if the income share of capital is high. Assuming that $f(k) = k^b$, the $r(k_{t+1})$ function is not monotone when $l < b$. The non-monotone feature is not specific to this model, but arises from broad classes of agency models of corporate finance, including the costly-state-verification approach *a la* Townsend (1979).

5. Equilibria in a World Economy with Financial Market Integration

Having investigated the case of autarky, in this section we study a world economy that consists of two countries that are inherently identical except for the initial stock of capital stock. We consider the following integration. The final good is tradable, and thus borrowing and lending of the final good are allowed across countries. Capital goods are not tradable from one country to another so that capital goods that are produced in country i $(i = N, S)$ have to be used in the final-good sector of country i. Labor is immobile across countries. We impose a simple assumption on enforcement technology. Assume that enforcement does not favor domestic investors, but deals with domestic and foreign investors equally.[9] Letting k_t^i denote the capital stock in country i in period t, we assume without loss of generality that $k_0^S \leq k_0^N$.

As a counterpart of (A2), we impose the following assumption.

(A3) $$aW(k_t^S) + aW(k_t^N) < 2(1-a)$$

for all $t's$. The total supply of funds is always smaller than the total demand for funds. Under (A3), the interest rate of each country should evolve as

(6) $$r_{t+1}^i = r(k_{t+1}^i) \equiv \begin{cases} lRf'(k_{t+1}^i) + W(k_{t+1}^i) & if \quad k_{t+1}^i < K(l), \\ Rf'(k_{t+1}^i) & if \quad k_{t+1}^i \geq K(l), \end{cases}$$

$(i = N, S)$. The market clearing in the international financial market requires the aggregate investments to be equal to the aggregate savings, given by

(7) $$k_{t+1}^N + k_{t+1}^S = RaW(k_t^N) + RaW(k_t^S).$$

In the world financial market, people of both countries face an identical world interest rate r_{t+1} so that we should have

(8) $$r_{t+1}^N = r_{t+1}^S = r_{t+1}.$$

Accordingly, any of the equilibria with an integrated financial market is described as a sequence $\{k_t^N, k_t^S, r_t\}_{t=0}^{\infty}$, satisfying (6), (7), and (8), given $k_0^N (>0)$ and $k_0^S (>0)$.

One remarkable departure of this paper from Boyd and Smith (1997) and Matsuyama (2004) is that our model can be reduced to a one-dimensional map of k_t^S or k_t^N.The analysis of a one-dimensional map allows us to conduct a global analysis.

The dynamic behavior differs according to whether $r(.)$ is monotone or not.. We first study the case when $r(.)$ is monotone decreasing. Since $r(.)$ is not differentiable at $K(l)$, we

[9] Actually, enforceability may depend on several dimensions, including the nationality, the distinction between traded and non-traded goods, the distance between countries, and the state of the economy. Enforceability is exogenous in our model, while as motivated by sovereign debt

have to analyze the correspondence from k_{t+1}^S to k_{t+1}^N by separating the whole interval into $[0, K(l))$ and $[K(l),+\infty)$, for k_{t+1}^S and k_{t+1}^N, respectively. It follows from the monotonicity and the symmetric property that (8) is described as

$lRf'(k_{t+1}^N)+W(k_{t+1}^N) = lRf'(k_{t+1}^S)+W(k_{t+1}^S)$ for $k_{t+1}^N < K(l)$ *and* $k_{t+1}^S < K(l)$, and

$Rf'(k_{t+1}^N) = Rf'(k_{t+1}^S)$ for $k_{t+1}^N \geq K(l)$ *and* $k_{t+1}^S \geq K(l)$. It is straightforward to derive

(9) $$k_{t+1}^N = k_{t+1}^S,$$

for any $k_{t+1}^S \in [0,+\infty)$ and $k_{t+1}^N \in [0,+\infty)$. Therefore, it follows from (7) and (9) that $k_{t+1}^i (i = N, S)$ evolves simply as $k_{t+1}^i = RaW(k_t^i)$, with $k_t^N = k_t^S$. A steady state is characterized by a pair $(k^N, k^S) = (k^*, k^*)$, satisfying $k^N = k^S$ and the constraint for the fund allocation between two countries

(10) $$k^N + k^S = RaW(k^N) + RaW(k^S).$$

Figure 4-0 illustrates the dynamic configuration in the (k_{t+1}^S, k_{t+1}^N) space. The curve going through $(k^*,0), E$, and $(0, k^*)$ represents the steady state relationship for the market clearing in the world financial market, given by (10). While, PQ represents the pair (k_{t+1}^S, k_{t+1}^N) that satisfies (7) given k_t^S and k_t^N, the temporal condition for the market clearing in the world financial market.

Assume that levels of capital of two countries initially lie at A_0 at period 0. On integration of the financial markets, the equilibrium jumps at period 1 to A, an intersection of the 45 degree line and the PQ line. Here a pair of capital stocks that will be attained under autarky, denoted A', is described so as to lie on PQ, satisfying $k_1^N + k_1^S = RaW(k_0^N) + RaW(k_0^S)$.

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consideration, enforceability may be controlled by the government.

Figure 4-0A illustrates the configuration of the world financial market, which is reminiscent of MacDougall (1960), Kemp (1962), and Hamada (1966). The length of interval from O_N to O_S stands for the initial stock of capital available to two countries, R times of sum of the aggregate savings in two countries, $Ra\{W(k_0^N)+W(k_0^S)\}$. Thus, from (7), any point in horizontal axis stands for the allocation of capital between two countries, where the length of interval from $O_N (O_S)$ to the point stands for the capital in country N(S) at period 1, k_1^N (k_1^S). The vertical axis stands for the interest rate realized in each country given $r(k_1^i)$ $(i = N, S)$. The function $r(k_1^N)$ is illustrated from O_N and $r(k_1^S)$ from O_S, respectively, and both functions are illustrated in symmetry. Each function $r(k_1^i)$ is illustrated to be decreasing.

We have to specify the price adjustment rule in the world financial market. We assume the Marshallian adjustment rule in which the demand and the supply changes according to the excess demand price. Investors rationally anticipate real interest rates of the two countries that will be realized if the autarkic state A' would prevail, and find the northern interest rate to be lower than the southern one, as implied by $r(k_1^{N,A}) < r(k_1^{S,A})$, where $k_1^{i,A}$ $(i = N, S)$ denotes the level of capital that attains under autarky. On opening the financial market, northern people keep investing their funds home, while southern people invest in the other country. Capital flows out of country N into country S, until the interest rate differential disappears. The equilibrium is achieved at A

After having reached A, the equilibrium converges to the steady state E. The steady state is symmetric in the sense that stocks of capital are the same between countries. The long-run consequence is irrespective of the initial cross-country distribution of capital. If financial markets are integrated between both countries, the equilibrium would converge to a unique steady state with equality no matter how the distribution of capital stock in the event of integration is. Note that Figure 4-0A is depicted so that both countries face financial constraints at the steady state, but the equilibrium may or may not involve the binding financial constraint, depending on parameter values.

We turn to the case when $r(.)$ is not monotone decreasing. As Figure 3-2 illustrates, $r(.)$ is first decreasing for $0 \leq k_{t+1}^i < lR$, later increasing for $lR \leq k_{t+1}^i < K(l)$, and finally decreasing for $K(l) \leq k_{t+1}^i$. In addition, $r(.)$ is not differentiable at $K(l)$. In order to apply the implicit function theorem, we divide the whole interval $[0, +\infty)$ into $[0, lR)$, $[lR, K(l))$, and $[K(l), +\infty)$. Thus the correspondence from k_{t+1}^S to k_{t+1}^N turns out to be composed of nine functions, each of which is continuous and differentiable on its distinct interval. The functions can be summarized as

(11) $$k_{t+1}^N = \Psi(k_{t+1}^S) = \begin{cases} k_{t+1}^S & \text{if} \quad k_{t+1}^N < lR \quad \text{and} \quad k_{t+1}^S < lR \\ m(k_{t+1}^S) & \text{if} \quad lR \leq k_{t+1}^N < K(l) \quad \text{and} \quad k_{t+1}^S < lR \\ f(k_{t+1}^S) & \text{if} \quad K(l) \leq k_{t+1}^N \quad \text{and} \quad k_{t+1}^S < lR \\ m(k_{t+1}^S) & \text{if} \quad k_{t+1}^N < lR \quad \text{and} \quad lR \leq k_{t+1}^S < K(l) \\ k_{t+1}^S & \text{if} \quad lR \leq k_{t+1}^N < K(l) \quad \text{and} \quad lR \leq k_{t+1}^S < K(l) \\ f(k_{t+1}^S) & \text{if} \quad K(l) \leq k_{t+1}^N \quad \text{and} \quad lR \leq k_{t+1}^S < K(l) \\ j_1(k_{t+1}^S) & \text{if} \quad k_{t+1}^N < lR \quad \text{and} \quad K(l) \leq k_{t+1}^S \\ j_2(k_{t+1}^S) & \text{if} \quad lR \leq k_{t+1}^N < K(l) \quad \text{and} \quad K(l) \leq k_{t+1}^S \\ k_{t+1}^S & \text{if} \quad K(l) \leq k_{t+1}^N \quad \text{and} \quad K(l) \leq k_{t+1}^S \end{cases},$$

where $m(k_{t+1}^S)$ is decreasing, $f(k_{t+1}^S)$ is increasing, $j_1(k_{t+1}^S)$ is increasing, and $j_2(k_{t+1}^S)$ is decreasing, with $m(lR) = lR$,, $\lim_{k_{t+1}^S \to K(l)-} m(k_{t+1}^S) = \lim_{k_{t+1}^S \to K(l)+} j_1(k_{t+1}^S) = j_1(K(l))$,

$\lim_{k_{t+1}^N \to K(l)-} m^{-1}(k_{t+1}^N) = \lim_{k_{t+1}^N \to K(l)+} f^{-1}(k_{t+1}^N) = K(l)$,

$\lim_{k_{t+1}^N \to lR-} j_1^{-1}(k_{t+1}^N) = \lim_{k_{t+1}^N \to lR+} j_2^{-1}(k_{t+1}^N) = f(lR)$, $\lim_{k_{t+1}^S \to K(l)-} f(k_{t+1}^S) = K(l)$, $f(K(l)) = K(l)$,

and $\lim_{k_{t+1}^S \to K(l)+} j_2(k_{t+1}^S) = j_2(K(l)) = K(l)$ $j_2(K(l)) = K(l)$. The derivation is left to Appendix. Figure 4-1 illustrates the correspondence $\Psi(.)$, which is composed of both the 45 degree line and the *ellipse*.

Any of steady states is defined as a pair $\{k^N, k^S\}$, satisfying (10) and

(12) $$k^N = \Psi(k^S).$$

If $\Psi(k^S) = k^S$, a steady state is a symmetric equilibrium, while otherwise, a steady state is an asymmetric equilibrium. It finally follows from (7) and (11) that the system can be reduced to a one-dimensional map of k_t^S, which is given by

(13) $$k_{t+1}^S + \Psi(k_{t+1}^S) = Ra\{W(k_t^S) + W(\Psi(k_t^S))\}.$$

Then there exists a correspondence from k_t^S to k_{t+1}^S that is continuous and differentiable except for *at most countable points*:

(14) $$k_{t+1}^S = \Phi(k_t^S),$$

with

(15) $$\Phi'(k_t^s) \equiv \frac{dk_{t+1}^s}{dk_t^s} = Ra\,\frac{W'(k_t^s) + W'(\Psi(k_t^s))\Psi'(k_t^s)}{1+\Psi'(k_{t+1}^s)},$$

with $\Psi'(k_{t+1}^s) \neq -1$, where $\Psi'(.) = f'(.)$ for $(k_{t+1}^S, k_{t+1}^N) \in [0, K(l)) \times [K(l), \infty)$,

$\Psi'(.) = m'(.)$ for $(k_{t+1}^S, k_{t+1}^N) \in [0, lR) \times [lR, K(l))$ or $\in [lR, K(l)) \times [0, lR)$,

$\Psi'(.) = j_1'(.)$ for $(k_{t+1}^S, k_{t+1}^N) \in [K(l), \infty) \times [0, lR)$, and $\Psi'(.) = j_2'(.)$ for

$(k_{t+1}^S, k_{t+1}^N) \in [K(l), \infty) \times [lR, K(l))$.

We should comment on a possible singular case of $\Psi'(k_{t+1}^S) = -1$. We cannot generally rule out a case when the system traverses $\Psi'(.) = -1$ in the asymmetric region with $k_{t+1}^N \neq k_{t+1}^S$. In order to focus on the analysis with well-behaved equilibria, we impose the following two technical conditions.

Condition 1: $m'(k_{t+1}^S) < -1$ if $k_{t+1}^S < lR$ and $m'(k_{t+1}^S) > -1$ if $k_{t+1}^S > lR$

Condition 2: $f'(.) > -1$ and $j_2'(.) < -1$

Condition 1 is satisfied when the production function is the Cobb-Douglas form. Condition 2 is satisfied for plausible parameters when the production function is the Cobb-Douglas form. The proof is technical and available on request. Specifically, we have $m'(k_{t+1}^S) = -1$ at (Rl, Rl). But on the 45 degree line ($k_{t+1}^N = k_{t+1}^S$), k_t^S evolves according to $k_{t+1}^S = RaW(k_t^S)$ [10] so that the singular problem is innocent at the symmetric region.

These two conditions are useful to investigate dynamic properties. If $\Psi(.)$ is increasing, $\Phi(.)$ is increasing so that the dynamic path shows either a monotone convergence to a steady state or a monotone divergence from the steady state. If $\Psi(.)$ is decreasing but its derivative is greater than -1, as implied by Condition 2, $\Phi'(k_t^s) = Ra\dfrac{W'(k_t^s) + W'(\Psi(k_t^s))\Psi'(k_t^s)}{1 + \Psi'(k_{t+1}^s)}$ should be positive since $W'(k_t^S) > W'(\Psi(k_t^S))$ for $k_t^S < k_t^N = \Psi(k_t^S)$ so that $\Phi(.)$ is increasing. However, if $\Psi(.)$ is decreasing with the derivative of less than -1 , as implied by Condition 1, $\Phi(.)$ may be increasing or decreasing. If $\Phi(.)$ is decreasing around a steady state, the dynamic path will fluctuate through cycles. Hereafter we use "monotone" in the sense that the system does not show cyclical behavior.

Let k^S denote any steady state to satisfy $k^S = \Phi(k^S)$. We obtain the following.

Proposition 1

Let $\tilde{k}$ denote a smallest steady state. Under Conditions 1 and 2, if $\Phi'(k) > 0$ for any $k_t^S \in (\tilde{k} - e, +\infty)$ with any small e, any dynamic path in the system is expressed as a monotone convergence or divergence from any steady state.

Proof: We prove this by illustrating the case with three steady states. For other cases, the procedure for the proof is essentially the same and deleted. First, we prove the simple case of

[10] Using $\lim_{k_{t+1}^S \to Rl} m(k_{t+1}^S) = Rl$, we have $\lim_{k_{t+1}^S \to Rl} m'(k_{t+1}^S) = \lim_{k_{t+1}^S \to Rl} \{1/m'(k_{t+1}^S)\}$, where l'Hopital's rule is used for equality. Given that $m(.)$ is decreasing, $\lim_{k_{t+1}^S \to Rl} m'(k_{t+1}^S) = -1$.

$k_t^N = k_t^S$. If $k_t^N = k_t^S$, the system can be reduced to $k_{t+1}^i = RaW(k_t^i)(i = N, S)$ so that k_t^S (or k_t^N) converges monotonically to a symmetric steady state E.

Next we prove the significant case of $k_t^N \neq k_t^S$ in three steps.

(Step 1): For any $k_t^S \in \Omega_1 \equiv \{k_t^S \mid K(l) \leq f(k_t^S)$ and $k_t^S < K(l)\}$, $\Phi(.)$ is bounded, so that $\Phi(.)$ is continuous. We next show that as $k_t^S \to K(l)$, $k_{t+1}^S < K(l)$. As $k_t^S \to K(l)$, $k_{t+1}^S + k_{t+1}^N = Ra\{W(k_t^S) + W(f(k_t^S))\} \to 2RaW(K(l))$ since $\lim_{k_t^S \to K(l)} f(k_t^S) = K(l)$. It follows from (A1) and $K(l) > k^*$ that $2RaW(K(l)) < 2K(l)$, which implies that as $k_t^s \to K(l)$, $k_{t+1}^S + k_{t+1}^N < 2K(l)$. Then there exists a pair (k_{t+1}^S, k_{t+1}^N) with $k_{t+1}^N > k_{t+1}^S$ which satisfies $k_{t+1}^N = \Psi(k_{t+1}^S)$ and $k_{t+1}^S + k_{t+1}^N < 2K(l)$. It is obvious from $k_{t+1}^S + k_{t+1}^N < 2K(l)$ and $k_{t+1}^N > k_{t+1}^S$ that $k_{t+1}^S < K(l)$ should be met.[11] In Figure 4-2A, there is no steady state in Ω_1. Thus, when $\Phi(.)$ is continuous and increasing, as depicted in Figure 4-2D, $\Phi(.)$ has to be below the 45 degree line over this interval so that k_t^S is monotone decreasing.

(Step 2): For any $k_t^S \in \Omega_2 \equiv \{k_t^S \mid lR < k_t^S < K(l)$ and $m(k_t^S) < lR\}$ or $\in \Omega_3 \equiv \{k_t^S \mid K(l) \leq k_t^S$ and $j_1(k_t^S) < lR\}$, $\Phi'(.)$ is bounded so that $\Phi(.)$ is continuous. As $k_t^S \to lR$, $k_{t+1}^S + k_{t+1}^N = Ra\{W(k_t^S) + W(m(k_t^S))\} \to 2RaW(lR)$ since $\lim_{k_t^S \to lR} m(k_t^S) = lR$. It follows from (A1) and $lR < k^*$ and (A1) that $2RaW(lR) > 2lR$,

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[11] If $k_t^N > k_t^S$ and there exist multiple pairs (k_{t+1}^S, k_{t+1}^N) which satisfy (7) and (11), the selected equilibrium will be a pair (k_{t+1}^S, k_{t+1}^N) which satisfies $k_{t+1}^N > k_{t+1}^S$. For the discussion on the equilibrium selection, see section 6.

which implies that as $k_t^S \to lR$, $k_{t+1}^S + k_{t+1}^N > 2lR$. Then there exists a pair (k_{t+1}^S, k_{t+1}^N) with $k_{t+1}^N < k_{t+1}^S$ which satisfies $k_{t+1}^N = \Psi(k_{t+1}^S)$ and $k_{t+1}^S + k_{t+1}^N > 2lR$. It is obvious from $k_{t+1}^S + k_{t+1}^N > 2lR$ and $k_{t+1}^N < k_{t+1}^S$ that $k_{t+1}^S > lR$ is met. In Figure 4-2A, there is a unique steady state in Ω_2, denoted by G, and no steady state in Ω_3. Thus, when $\Phi(.)$ is continuous and increasing, as depicted in Figure 4-2E, $\Phi(.)$ has to cross the 45 degree line only once at $k_t^S = k_G$ from above. Let k_G denote the level of capital stock in country S corresponding to at the steady state G. Then as depicted in Figure 4-2E, k_t^S is monotone increasing if $k_t^S < k_G$ and monotone decreasing if $k_t^S > k_G$.

The dynamic behavior of k_t^S over this interval is a mirror image of that of k_t^N for $k_t^N \in \{k_t^N \mid lR < k_t^N < K(l)$ and $m^{-1}(k_t^N) < lR\}$ or $\in \{k_t^N \mid K(l) \le k_t^N$ and $f^{-1}(k_t^N) < lR\}$. Analogously from the above argument, k_t^N converges monotonically to F over this interval.

(Step 3): k_t^S should be related to k_t^N through (11). For $k_t^S \in \{k_t^S \mid k_t^S < lR$ and $K(l) \le f(k_t^S)\}$, k_t^S should be monotone decreasing, and for $k_t^S \in \{k_t^S \mid k_t^S < lR$ and $lR < m(k_t^S) < K(l)\}$, k_t^S should converge monotonically to k_F.Q.E.D.

We distinguish equilibria between four different cases roughly according to the number of steady states. Figure 4-2A illustrates one typical configuration of three steady state equilibria. This case arises if $lR < k^* < K(l)$ so that all countries are borrowing-constrained in the symmetric steady state.[12] Also in the asymmetric steady states all countries are

[12] If $k^* < lR$, the analysis is simple. Then there exists a unique steady state that is globally stable. This case arises if either a or A is small relative to lR.

borrowing-constrained. This case corresponds to the case studied by Boyd and Smith (1997). Around F, $\Psi(.)$ is decreasing with the derivative of less than -1 so that $\Phi'(.)$ may be positive or negative. The equilibrium around any of asymmetric steady states may exhibit cycles. Figure 4-2H depicts the case when $\Phi(.)$ is decreasing at k_F so that k_t^s fluctuates though cycles over the interval $[\Phi^2(k_c), \Phi(k_c)]$. There necessarily exists a 2-period cycle around the steady state.[13] [14] Any dynamical path lies on the 45 degree line or goes along the ellipse. Anyway, any path on the ellipse arrives at either of near the asymmetric steady states, but never to arrive at the symmetric one.

We have another configuration of three steady state equilibria [Figure is not illustrated]. In the asymmetric steady state poor country is borrowing-constrained, but the rich does not, with $k^S < lR < K(l) < k^N$. This case corresponds to the case investigated by Matsuyama (2004) as a "symmetry-breaking" case. Around F, $\Psi(.)$ is increasing so that $\Phi(.)$ is increasing. Under Conditions 1 and 2, any dynamic path in the system is expressed as a monotone behavior, as demonstrated in Proposition 1. The two asymmetric steady states, F and G, are stable, and the unique symmetric steady state, E, is also stable.

Figure 4-2B illustrates five steady states equilibria. This case arises if $K(l) < k^*$ so that all countries are borrowing-unconstrained in the symmetric steady state. In the asymmetric steady state poor country is borrowing-constrained, but the rich does not, with $k^S < lR < K(l) < k^N$.

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[13] The 2-period cycle may be stable or unstable. , depending on the .One method for characterizing global properties of the 2-period cycle is to check Schwarzian derivative. If the Schwarzian derivative is negative, the cycle is stable and has several nice properties, but the sign is indeterminate.

[14] Demonstrating the existence of any other higher-order cycles will be complicated and we do not pursue in this paper. Matsuyama (1999) develops a model of growth through cycles and investigates the possibility of higher-order cycles than 2-period cycle. Grandmont (1986) provides an accessible review of chaos.

[17] Assuming that $f(k) = k^b$, the $\Phi(.)$ function is increasing over the relevant interval so that the system is monotone if l >0.04 for $b = 0.3$ and if l >0.11 for $b = 0.4$ given that the smallest (net) interest rate is zero.

Under Conditions 1 and 2, any dynamic path in the system is expressed as a monotone behavior, as demonstrated in Proposition 1. The symmetric steady state, E, is stable. Noteworthy, among four asymmetric steady states, F and G, are stable , while the other two, H, and J, are unstable. The steady state H (or J) is a threshold across which equilibria diverges to different stable steady states. The equilibrium that lies at the left of H converges to the asymmetric steady state F, while the equilibrium that lies at the right of H will finally converge to the symmetric steady state E. The monotone dynamic behavior requires that l is not too small.[17]

Figure 4-2C illustrates the case with a unique steady state case. This case arises if $K(l) < k^*$. But the dynamic process is far different from the case when the $r(.)$ function is monotone, as depicted in Figure 4-0. There are multiple paths that converges to the steady state, E, One path is on the 45 degree line, while others on the ellipse. The path on the ellipse exhibits a perverse behavior. First, k_t^S decreases while k_t^N increases, later k_t^S and k_t^N both increase, and finally k_t^S increases while k_t^N decreases.

Different configurations of equilibria emerge, depending on the world-market-clearing condition at the steady state (10) and the ellipse (11), both of which depend on parameter values. The world-market-clearing condition at the steady state (10) depends on the TFP A, the productivity of the project R and the population of investors a, a direct measure of the global saving.[19] As either of these parameters gets greater, (10) tends to expand outward from

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[19] Consider the Diamond's standard overlapping generation economy with two-period-lived agents. Assume that the preference of an agent of country $i(=N,S)$ is represented by $\log C_t^y + b_i \log C_{t+1}^o$, where $C_t^y\,(C_{t+1}^o)$ denote consumption when they are young (old), and b_i is the subjective discount factor of a person of a country i. Then the saving rate of a country i is $b_i/(1+b_i)$. The aggregate fund allocation in the world economy is described as $k_{t+1}^N + k_{t+1}^S = \frac{b_N}{1+b_N} W(k_t^N) + \frac{b_S}{1+b_S} W(k_t^S)$. Comparing this equation and (7), it is obvious to see that a is the direct measure of a country's saving rate.

the origin because an increase in either A, R, or a makes the aggregate capital stock available to both countries greater. The ellipse (11) depends on R and the fraction of confiscation of the revenue by investors l, a measure of financial development.[20] As l becomes smaller, the ellipse get greater with lR getting smaller and $K(l)$ getting greater. Conversely, as l becomes greater, the ellipse get smaller, and finally disappears for the threshold $\hat{l}$.[21]

It turns out that as either A, l (the measure of contract enforcement) or a (the measure of global savings) is small, the equilibria with three steady states are more likely to occur, as depicted in Figure 4-2A. As either of the three parameters increases or all, the equilibria with five steady states are likely to emerge, as depicted in Figure 4-2B. Finally, if all parameters increase further, the equilibria with a unique steady state are more likely to occur, as depicted in Figure 4-2C.

6. Global Analysis and Financial Integration

Having studied equilibrium dynamics of the world economy with financial integration, we turn to which of the equilibria is selected and in which direction capital flows along the equilibrium path. As in the previous section, we focus on the case when Conditions 1 and 2 are satisfied.

Figure 5-2A illustrates the case with three steady states. This case is more likely when the TFP is small, institutions for contract enforcement are less developed, and the world savings

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[20] If the TFP is taken into account, the aggregate saving-investment relation is described as $k_{t+1}^N + k_{t+1}^S = RaA_tW(k_t^N) + RaA_tW(k_t^S)$, while the real interest rate is expressed as $r_{t+1}^i = r(k_{t+1}^i) \equiv \begin{cases} lRA_t f'(k_{t+1}^i) + A_tW(k_{t+1}^i) & if \quad k_{t+1}^i < K(l), \\ RA_t f'(k_{t+1}^i) & if \quad k_{t+1}^i \geq K(l), \end{cases}$. It is obvious from the latter equation that the no-arbitrage condition is independent of A.

[21] The effect of an increase in R on (11) is ambiguous; an increase in R increases not only lR but also $K(l)$.

[24] Note that as long as Condition 1 and 2 are satisfied, there are at most three pairs which satisfy (7) and (11).

rate is low. Assume that both countries open capital accounts at some period T in the developing stage when k_T^N and k_T^S $(k_T^N > k_T^S)$ satisfy $2lR < Ra\{W(k_T^N)+W(k_T^S)\} < 2K(l)$. These inequalities say that the PQ line representing the market clearing in the world market, (7), lies at the northeast of (lR, lR) and at the southwest of $(K(l), K(l))$. At period $T+1$ there exist three intersections between the PQ line and (11). Two of the intersections, B_1 and B_3, lie at the intersection of PQ and the locus $k_{t+1}^N = m(k_{t+1}^S)$, while the rest of them, B_2, lies at the intersection of PQ and the 45 degree line.

Now look at B' on PQ that represents the levels of capital that would be attained under autarky at period $T+1$. Note that the point B' is inside the ellipse. The corresponding configuration of the world financial market is depicted in Figure 5-2B, where there are three intersections. The three intersections of $r(k_{T+1}^N)$ and $r(k_{T+1}^S)$ are denoted as B_1, B_2 and B_3, respectively, and each of them corresponds to the same symbol in Figure 5-2A.[24]

At the autarkic state B', investors will rationally anticipate the northern interest rate to be higher than the southern one to meet $r(k_{T+1}^{N,A}) > r(k_{T+1}^{S,A})$ since $k_{t+1}^{N,A} > m(k_{t+1}^{S,A})$. Following the Marshallian adjustment rule, capital flows out of country S into country N, until the interest rate differential disappears. Thus, the equilibrium is achieved at B_1 at period T+1. So long as autarkic economies reach somewhere on PQ with the northwest of the 45 degree line, the integrated equilibrium is achieved at B_1. Under this rule, the symmetric equilibrium B_2 is unstable, and B_3 is locally stable.[25] In Figure 5-2A, after having arrived at B_1, the equilibrium approaches to an asymmetric steady state F along $k_{t+1}^N = m(k_{t+1}^S)$. In the asymmetric steady state F, capital flows from country S country N.[26] In the asymmetric

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[25] Although we have argued that the equilibrium is settled down to B_1, we may not exclude the possibility for either B_2 or B_3 to be chosen as equilibria if an alternative adjustment rule would be specified. If investors react perversely to the interest rate differential, B_2 or B_3 might be viable as equilibrium accompanied by sudden capital reversal, which will be attributed not to fundamental reasons but to sunspot or confidence in the market. Some perturbation in confidence will lead to a sudden and great reversal of capital flows. In the case of Asian financial crisis, the collapse of confidence in the market triggered the reversal of capital.

[26] This result follows from (A1) and the fact the level of capital in country N is larger than k^*

steady state F, the marginal product of capital in country S is higher than the other, but the greater agency cost in the southern financial market deters the inflow of capital to country S. One answer to Lucas (1990) paradox is that the difference in the efficiency in the financial market makes it impossible for returns to capital to be equalized between countries. However, it is not easy to understand the direction of capital flows on the equilibrium path. In fact, we obtain the following lemma.

Lemma 1

(a) Suppose that (7) intersects with (11) at $(k_{t+1}^S, k_{t+1}^N) \in [0, lR) \times [lR, K(l))$. If an equilibrium pair of autarkic economies $(k_{t+1}^{S,A}, k_{t+1}^{N,A})$ satisfies $k_{t+1}^{N,A} > (<) m(k_{t+1}^{S,A})$, the equilibrium interest rates of the two countries satisfy $r(k_{t+1}^{N,A}) > (<) r(k_{t+1}^{S,A})$ under autarky.

(b) Suppose that (7) intersects with (11) at $(k_{t+1}^S, k_{t+1}^N) \in [0, K(l)) \times [K(l), \infty)$. If an autarkic pair $(k_{t+1}^{S,A}, k_{t+1}^{N,A})$ satisfies $k_{t+1}^{N,A} < (>) f(k_{t+1}^{S,A})$, the equilibrium interest rates satisfy $r(k_{t+1}^{N,A}) > (<) r(k_{t+1}^{S,A})$ under autarky.

Proof: We prove only part (a), because part (b) can be proved in a similar manner.

Pick up a pair (k_{t+1}^S, k_{t+1}^N) satisfying $k_{t+1}^N = m(k_{t+1}^S)$ and $k_{t+1}^S < lR \le k_{t+1}^N < K(l)$. We then pick up a pair under autarky $(k_{t+1}^{S,A}, k_{t+1}^{N,A})$ satisfying $k_{t+1}^{N,A} > m(k_{t+1}^{S,A})$, by increasing k_{t+1}^N, starting from (k_{t+1}^S, k_{t+1}^N), while fixing k_{t+1}^S. We have

$lRf'(k_{t+1}^N) + W(k_{t+1}^N) > lRf'(m(k_{t+1}^S)) + W(m(k_{t+1}^S)) = lRf'(k_{t+1}^S) + W(k_{t+1}^S)$, where the inequality arises from $k_{t+1}^{N,A} > m(k_{t+1}^{S,A})$ and the fact that $lRf'(.) + W(.)$ is increasing in k_{t+1}^N,

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while that in country S is smaller than k^* (e.g., Boyd and Smith (1997, Proposition 5)).

and the equality arises from the definition of $m(.)$ and $k_{t+1}^{S,A} = k_{t+1}^{S}$. It finally follows that

$r(k_{t+1}^{N,A}) > r(k_{t+1}^{S,A})$. Q.E.D.

Although we prove only the region of $k_t^N > k_t^S$, by symmetry we can develop a similar argument for the region of $k_t^N < k_t^S$. Hence we have the following corollary from Lemma 1 and the symmetric argument.

Corollary 1

If an autarkic pair $(k_{t+1}^{S,A}, k_{t+1}^{N,A})$ is inside (outside) the ellipse, equilibrium interest rates in two countries satisfy $r(k_{t+1}^{N,A}) > (<) r(k_{t+1}^{S,A})$ under autarky.

It is straightforward to have the following.

Proposition 2

Suppose that an equilibrium pair (k_t^S, k_t^N) satisfies $k_t^N = \Psi(k_t^S)$. Then if the state of would-be autarkic economies at period $t+1$, $(k_{t+1}^{S,A}, k_{t+1}^{N,A})$, is inside (outside) the ellipse, capital flows from country S (N) to country N(S).

The position of the would-be autarkic state at period $t+1$ relative to the ellipse determines the direction of capital flows at period $t+1$. When the levels of capital are not so large in all the countries, the autarkic state is likely to lie inside the ellipse, and capital is likely to flow out of country S to country N. Proposition 2 also implies that as long as (7) intersects with the ellipse, $k_{t+1}^N > k_{t+1}^S$ will be preserved.

Looking at Figure 5-2C, we see the direction of capital movement on the path to F. The arrows in solid line indicate where the state of would-be autarkic economies is located at next period given that the equilibrium is on the ellipse. When (k_t^S, k_t^N) lies at $[0, k^*)\times[0, k^*)$, the state of would-be autarkic economies at period $t+1$, $(k_{t+1}^{S,A}, k_{t+1}^{N,A})$, lies at the northeast of the current state,[27] while when (k_t^S, k_t^N) lies at $[0, k^*)\times[k^*, \infty)$, $(k_{t+1}^{S,A}, k_{t+1}^{N,A})$ lies at the southeast of the current state. It is obvious that all the arrows approach toward (k^*, k^*). In Figure 5-2C, all the arrows are described to be directed inside the ellipse at any point on $k_{t+1}^N = m(k_{t+1}^S)$ around F. Thus, Proposition 2 predicts that on almost all the path to the asymmetric steady state F, capital flows out of country S to country N.

If the integration occurs at the early developing stage, with k_T^N and k_T^S being so small to satisfy $Ra\{W(k_T^N)+W(k_T^S)\}<2lR$ or at the stage, with k_T^N and k_T^S being great to satisfy $2K(l)<Ra\{W(k_T^N)+W(k_T^S)\}$ (although the latter case is unlikely), the intersection is uniquely realized on the 45 degree line, and the equilibrium converges to the symmetric steady state E.[28] However, the symmetric steady state and the path on the 45 degree line are unstable in the following sense. In Figure 5-2B, the initial equilibrium is achieved at, for example, B_2, but for a small perturbation the financial market requires to be cleared at either of the asymmetric equilibria, B_1 and B_3 because $r(.)$ is upward sloping around B_2. Once the equilibrium diverges from the symmetric equilibrium, it never goes back, but eventually falls into any one of the asymmetric steady states. If a small perturbation is allowed for along the development path, the equilibrium eventually falls into one of the asymmetric steady states.

The symmetric steady state is seldom achieved; financial integration leads to an increased inequality of per-capita income between countries with capital flows from a poor to a wealthy

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[27] This is because (A1) implies that $k_t^i < k_{t+1}^{i,A} < k^*$ holds if $k_t^i < k^*$, while $k_t^i > k_{t+1}^{i,A} > k^*$ holds if $k_t^i > k^*$.

[28] If $2lR < Ra\{W(k_T^N)+W(k_T^S)\} < 2K(l)$ is satisfied and the stocks of capital are accidentally the same at the timing of integration, as shown in Figure 5-2A, the equilibrium at period t+1 is achieved at B_2, and converges to the symmetric steady state E.

country.[29] The "symmetric-breaking" behavior of the world financial market has already been extensively argued by many researches, including Boyd and Smith (1997), Sakuragawa and Hamada (2001) and Matsuyama (2004).

Figure 5-3A illustrates the case with five steady states. This case is more likely when the TFP is fairly high, institutions for contract enforcement are fairly developed, and the world savings rate is at the intermediate level. Now suppose, for example, that under autarky C' will be reached in period $T+1$ (note that $k_T^N > k_T^S$). Then there exist three intersections between the PQ line representing (7) and (11), denoted, C_1, C_2, and C_3. In this case, $r(k_{t+1}^{N,A}) > r(k_{t+1}^{S,A})$ holds (Lemma 1(b)), and hence at the instant of integration southern people have an incentive to invest in country N. Thus at period T+1 the equilibrium is achieved at C_1. Having arrived at C_1, the equilibrium converges to an asymmetric steady state F with a flight of capital from country S to country N. In Figure 5-3C, arrows are typically directed inside the ellipse at almost all the points on the path converging to F so that capital flows out of country S to country N on almost all the path to the asymmetric steady state F. When the world economy arrives at F, it turns out that the globalization leads to an increased inequality of per-capita income between countries.

However, we have a different consequence of financial integration in Figure 5-3A. Consider the case when both countries are sufficiently wealthy that the autarkic equilibrium reaches D'. At D', $r(k_{t+1}^{N,A}) > r(k_{t+1}^{S,A})$ holds (Lemma 1(b)), and hence at the instant of integration southern people have an incentive to invest in country N. The outflow of capital from country S to country N shifts the equilibrium to D_1. However, once the equilibrium arrives at D_1, the equilibrium goes on $k_{t+1}^N = f(k_{t+1}^S)$ toward I, and after passing through I converges to the symmetric steady state E. Figure 5-3C shows that on the path to I, arrows are directed outside the ellipse, and thus capital tends to flow from country N to country S. Figure 5-3B represents the configuration of the world financial market on the symmetric path. The aggregate

[29] Note that poor countries then receive the gain of possessing foreign assets so that the GNP gap is not so large as GDP.

world savings are now great so that the function $r(k_{t+1}^N)$ shifts to the left and the function $r(k_{t+1}^S)$ to the right, compared to Figure 5-2B. The intersection is unique and determined at I to allocate capital equally between countries.

The direction of capital flows differs along the developing stage, depending on the levels of capital stock of both countries. A look at non-monotone configuration of $r(k)$ illustrated in Figure 3-3 helps our understanding behind this mechanism. When the level of capital stock realized under autarky lies around lR, the corresponding interest rate tends to be lower than that of the other country, and thus the country's saving flows out abroad by openness. However, once the autarkic level of capital stock reaches the level near $K(l)$, the interest rate tends to be higher than that of the other, and thus the country enjoys capital inflows by openness.

To which steady state the world economy will converge depends on the distribution of the capital stock when integration occurs. From the above argument, if the PQ line goes across the western region relative to H on the steady-state saving-investment relation (10), the integration will move the equilibrium to an asymmetric steady state, while otherwise, the integration will eventually move the equilibrium to the symmetric steady state. Thus, as depicted in Figure 5-3D, the shaded region illustrates the pair of levels of autarkic capital stock under which the symmetric steady state is finally realized. The downward curve is a set of pairs (k_t^S, k_t^N) satisfying $k_H^N + k_H^S = RaW(k_t^N) + RaW(k_t^S)$, where (k_H^S, k_H^N) is the pair of levels of capital corresponding to H. If the levels of capital in both countries are high, the integration tends to move the world economy to the symmetric steady state with harmonized growth. Furthermore, given the aggregate capital of both countries constant, if the inequality is small, the integration tends to be successful.[30]

From the prospect of late-developing countries, the capital account liberalization promotes development only if some threshold level of income has already been attained. As has raised by many observers and government officials, including McKinnon (1991), this theoretical finding explains the important role of the timing of lifting capital account for late-developing countries.

[30] Since the curve with shade is convex to the origin, any PQ line is more likely to intersect with the

Late-developing countries can go on the successful path by delaying the timing of liberalization until arriving at some development stage.

Conditions for successful financial integration depend on the levels and the distribution of per-capita income across countries. Other things being equal, higher levels of per-capita income and more equal per-capita income are determinants of successful integration. Furthermore, several parameters, A (the TFP level), l (the measure of contract enforcement), or a (the measure of global savings) will affect the point H, and so the downward curve depicted in Figure 5-3D, and thus the period for successful integration. Other things being equal, a greater value of either A, l, or a makes the aggregate savings necessary to attain successful integration, $k_H^N + k_H^S = RaW(k_t^N) + RaW(k_t^S)$, smaller, and hence shorten the waiting time for successful financial integration.

Some empirical evidence is consistent with the theoretical finding. Chinn and Ito (2006) investigate whether financial openness leads to financial development using panel data covering 108 countries over the period of 1980-2000, and find that financial openness promotes development of domestic financial markets only if a threshold level of legal development has been attained. Bekaert et al, (2001) find that financial liberalization tends to promote economic growth and the effect is greater for countries with high education levels in their sample of 30 emerging countries.

Episodes also abound. It was only in the 1980s that the governments of fast-growing East Asian countries, including Japan, Korea, and Taiwan, lifted capital account completely. The Japanese government announced liberalization of capital control in 1979, but the Japanese liberalization was not genuine toward 1980s. In Taiwan, capital transactions were decontrolled only as recent as in 1987. In Korea, liberalization was gradually made toward 1980s, but restrictions on capital movements are not yet completely removed. Although capital controls are in principle aimed at restricting capital outflows, some countries experienced rapid and massive inflows of capital soon after the removal, including Italy, Spain, New Zealand, and Uruguay. Bartolini and Drazen (1997) provided these episodes in order to emphasize their signaling

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area of the shaded region with inequality being small.

argument of capital account liberalization as a commitment to policy reforms, but these countries may have attained the stage in which the reversal of capital flows will happen.

Finally, Figure 5-4A illustrates the case with the unique steady state. This case is more likely when the TFP is high, institutions for contract enforcement are well developed, and the world savings rate is high. Figure 5-4A illustrates the typical case showing that the early lifting capital accounts makes the period necessary to arrive at the steady state longer. To see this, suppose for example that two economies will reach at Z' in period $T+1$ if they are in autarky. At Z', $r(k_{T+1}^{N,A}) > r(k_{T+1}^{S,A})$ holds (Lemma 1(b)), and hence southern people have an incentive to lend to country N. The equilibrium at period T+1 is achieved at Z_1. Once the economy arrives at Z_1, it goes on $k_{t+1}^{N} = f(k_{t+1}^{S})$ until it reaches I, and afterward, on the 45 degree line, eventually converges to the symmetric steady state E. Figure 5-4B shows that on the path reaching I, capital flows first from country S to N, later reverses the direction of capital flows S; on all the points to path from I to E, capital flows cease.

By financial integration, the convergence to the symmetric steady state will occur surely, but will give rise to an increased inequality of nations in the transition. On early financial integration, the catching-up country first experiences capital outflows, later a reversal of capital flows, and finally capital inflows. The “trickle-down effect” from wealthy countries reverses the direction of capital flows. Once fast-growing countries have achieved some stage of development, the increased global savings leads to a decline in the world interest rate, which in turn, stimulates borrowing and investment of late-developing countries, thus promoting these countries to catch up with fast-developing countries.[31]

From the prospect of late-developing countries, a best strategy for faster development is to keep capital accounts closed until the world economy under autarky reaches the state around Z_2. Conditions for successful financial integration depend on the levels of per-capita income across countries, but now not the distribution. Further development of institutions for contract enforcement, captured by a smaller l, will make the ellipse smaller, and the period for successful integration is earlier.

[31] Drazen develops an alternative theory behind which the capital reversal happens.

The illustrated the pattern of capital flows is similar to the one of Latin American countries past twenty years. In the 1980s, a number of Latin American countries experienced capital outflows, involving capital flights, but in the 1990s, capital begins to go back to these countries. Some economists argue that the reversal of capital movement in this region arises from the decline in the world interest rate driven by the change in the global saving-investment relation not from any domestic institutional change.[32]

7. Trade Openness and "Order of Liberalization"

The problem of the optimal timing for lifting capital account has been traditionally argued in terms of the sequence of liberalization of first trade and secondly capital flows (e.g. McKinnon (1991)). Braun and Raddatz (2007) report that trade liberalization occurred before capital account liberalization in 68 of the 73 countries that liberalized any of these dimensions between 1970 and 2000. Chin and Ito (2006) find that trade liberalization becomes a precondition for capital account liberalization.

Although some observers address the sequential openness, the rational behind the sequential liberalization is not necessarily clear. According to the standard trade theory, trade openness and financial market openness should be, and the sequence is irrelevant. In order to investigate this problem, particularly to explore the possible implications for sequential opening, we extend the model to allow for capital goods to be tradable.

Assume that among capital goods produced by entrepreneurs, a fraction $m(0<m<1)$ of them are tradable, while the remaining are not so that some of capital goods produced in country i can be used in the final-good firm in country $j(i \neq j)$. Many capital goods appear to be nontraded (e.g., railways). Accordingly, the stock of capital financed and produced in country i at period t, denoted $\tilde{k}_t^i$, is distinguished from that available in country i at period t, denoted k_t^i. Letting m_t denote the amount of capital goods that are exported from country N

[32] See Calvo (1996), for example.

to S, by definition we have $k_t^N = \tilde{k}_t^N - m_t$ and $k_t^S = \tilde{k}_t^S + m_t$. Together with the definitions, the market clearing in the world financial market finally reduces to (7).[33] The trade of capital goods implies the reallocation of capital goods between countries, and thus represented by a shift on the same PQ line. Since the trade of capital goods is motivated by the difference in the price of the capital good, the amount of capital goods traded m_{t+1} satisfies $f'(k_{t+1}^N) = f'(k_{t+1}^S)$, or $m_{t+1} = m\tilde{k}_{t+1}^N$ if $f'(k_{t+1}^N) < f'(k_{t+1}^S)$. Additionally, the no-arbitrage condition $r(k_{t+1}^N) = r(k_{t+1}^S)$ should be met.

If the trade realizes the equality $f'(k_{t+1}^N) = f'(k_{t+1}^S)$, the trade openness brings the world economy on the symmetric path, whereas otherwise, it tends to shift the capital allocation on the PQ line in the asymmetric region. If the liberalization of capital accounts is made together, the trade openness never influences the development path. The allocation of capital goods made by trade is offset by capital movement. In the case of simultaneous liberalization, trade and capital flows are substitutable.

We next turn to the sequential openness. We find an interesting case in the case of five steady states. In Figure 6-1A, at period T two closed economies arrive at I' that is outside the curve with shadow, but moves along PQ from I' to I'' with trade openness and with no lift of capital control. Once the world economy have arrived at I'' that is inside the curve with shadow, lifting capital control brings the world economy finally on the path to the symmetric steady state E.

Developing countries can attain faster growth first by trade liberalization and secondly by capital account liberalization. The benefit of the sequential openness arises when the timing of lifting capital control matters for development. The sequential problem becomes an urgent policy issue only if some threshold level of financial development has been attained.[34]

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[33] Note that $\tilde{k}_{t+1}^N + \tilde{k}_{t+1}^S = k_{t+1}^N + k_{t+1}^S$.

[34] If technological spillovers through the flow of traded goods are also taken into account, developing countries are more likely to gain the benefit from sequential openness.

8. Conclusion

We have considered a global analysis of financial market integration, and found several important features that have not been obtained from the local analysis around steady states. There are a stable symmetric and stable asymmetric steady states, both of which may coexist, and out of steady states, there are interesting non-monotone behavior of the pattern of capital movement and development. The timing of integration that attains harmonized growth depends on several characteristics including the stage of economic and legal developments, income inequality, and global savings. The concept of optimal timing for liberalization allows us to explore conditions for successful integration.

This paper is extended in several directions. A slight modification of the model can explain the persistent current account imbalance between the U.S. and some Asian countries, so-called, "great imbalance". In the growing economy the capital account surplus (deficit) is a reflection of capital outflows that will arise in a setup of countries with different institutions for contract enforcement. A modeling of that kind might lead to a conclusion that if China progresses the capital account liberalization following the request of the U.S., the US-China imbalance becomes even greater with further outflows of capital from China.

The introduction of foreign direct investment (FDI) will be a promising avenue to extend this model. FDI may be identified as equity participation to the project associated with the transfer of the control right of the firm, that should be distinguished from "security investment" involving the participation to the project as debtholders. How the richer movement of capital including both security and direct investments will change the development path and contribute to successful integration is an interesting issue to be explored.

The difference in the pledgeability among goods will be an interesting avenue to be examined. The pledgeability may differ according to how technological change is embodied or whether the output is a traded or a non-traded good. Following the model, the parameter l might differ between the traded and non-traded goods, or among many goods that differ in the extent of embodiment of technological change.

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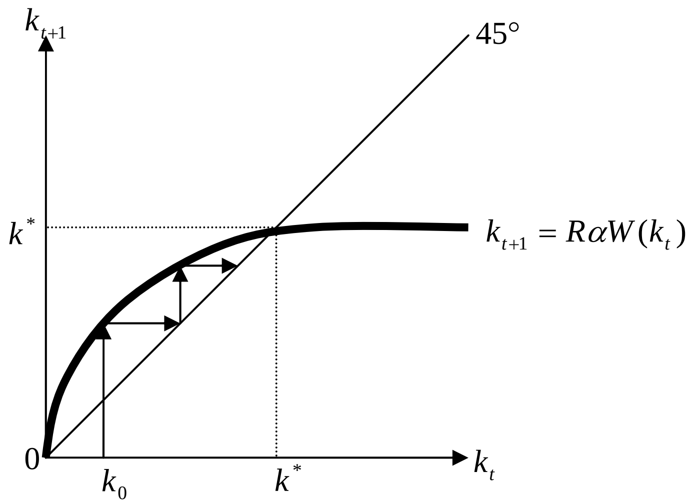
Figure 3-1



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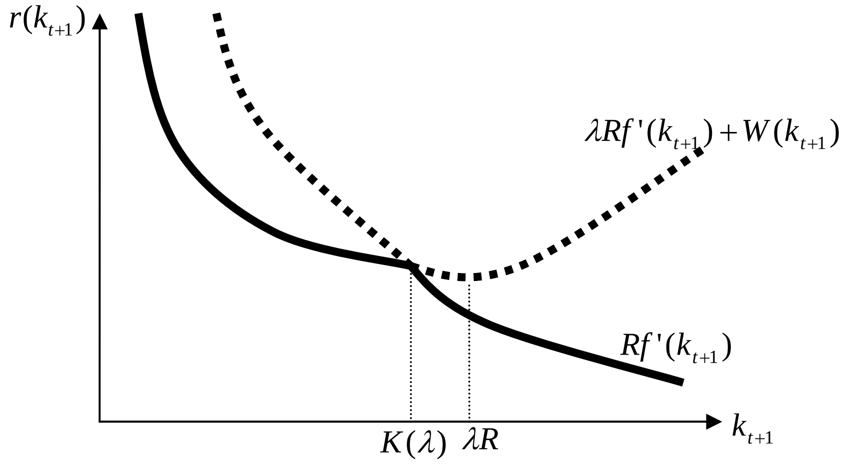

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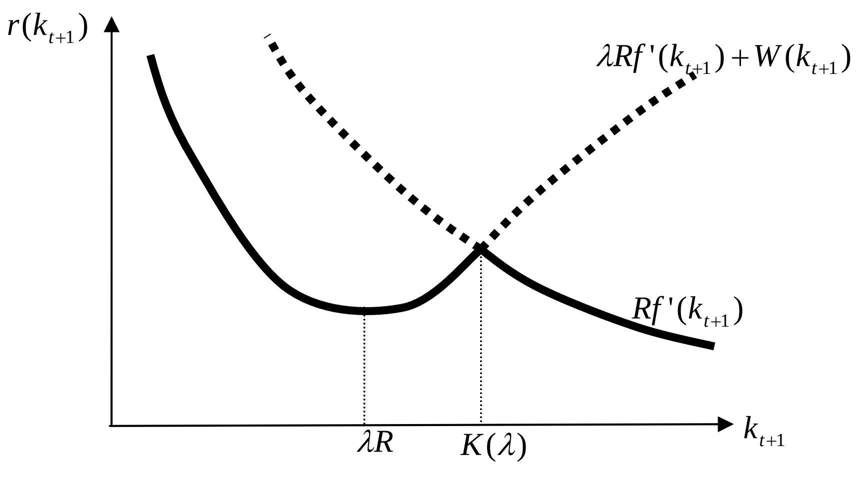

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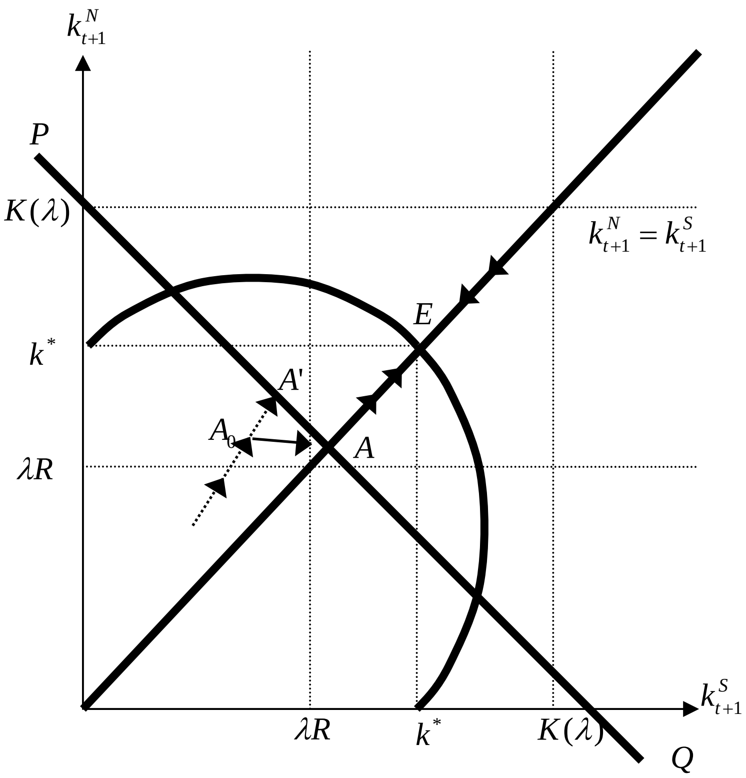


Figure 4-0A

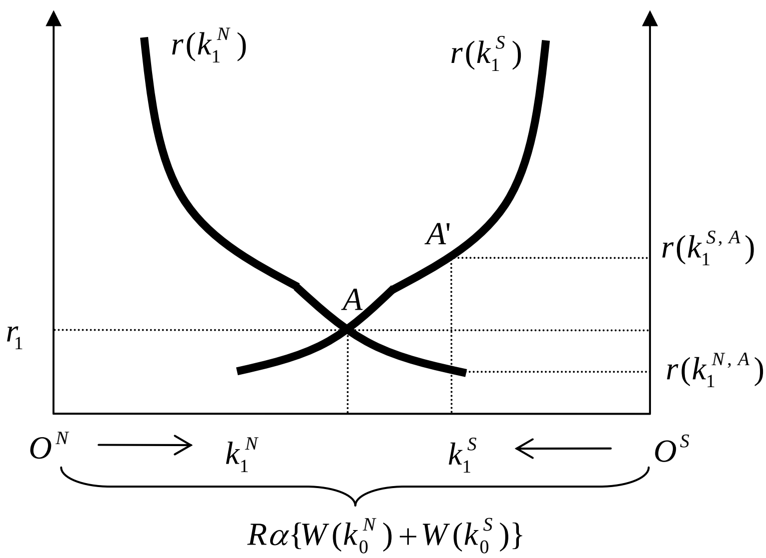
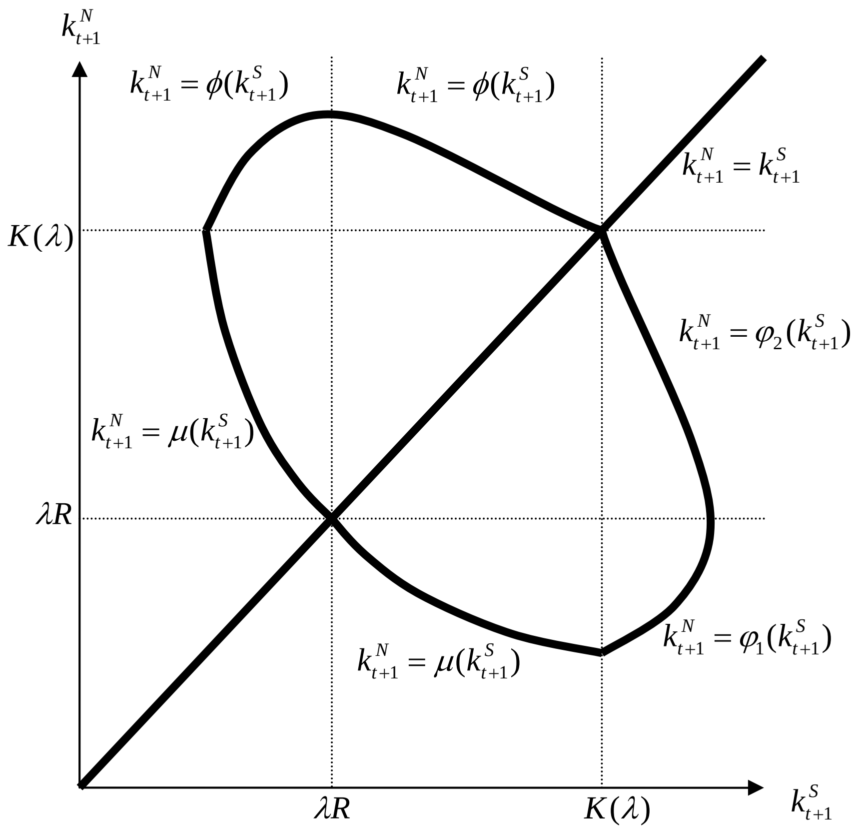


Figure 4-1



k_{t+1}^{N}
$k_{t+1}^{N} = \phi(k_{t+1}^{S})$
$k_{t+1}^{N} = \phi(k_{t+1}^{S})$
$k_{t+1}^{N} = k_{t+1}^{S}$
$K(\lambda)$
$k_{t+1}^{N} = \varphi_2(k_{t+1}^{S})$
$k_{t+1}^{N} = \mu(k_{t+1}^{S})$
λR
$k_{t+1}^{N} = \varphi_1(k_{t+1}^{S})$
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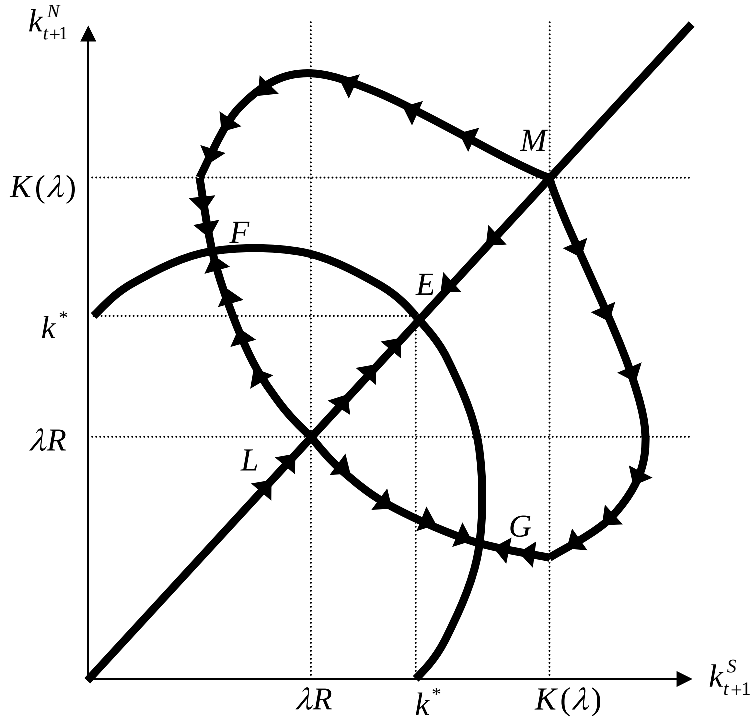
Figure 4-2A



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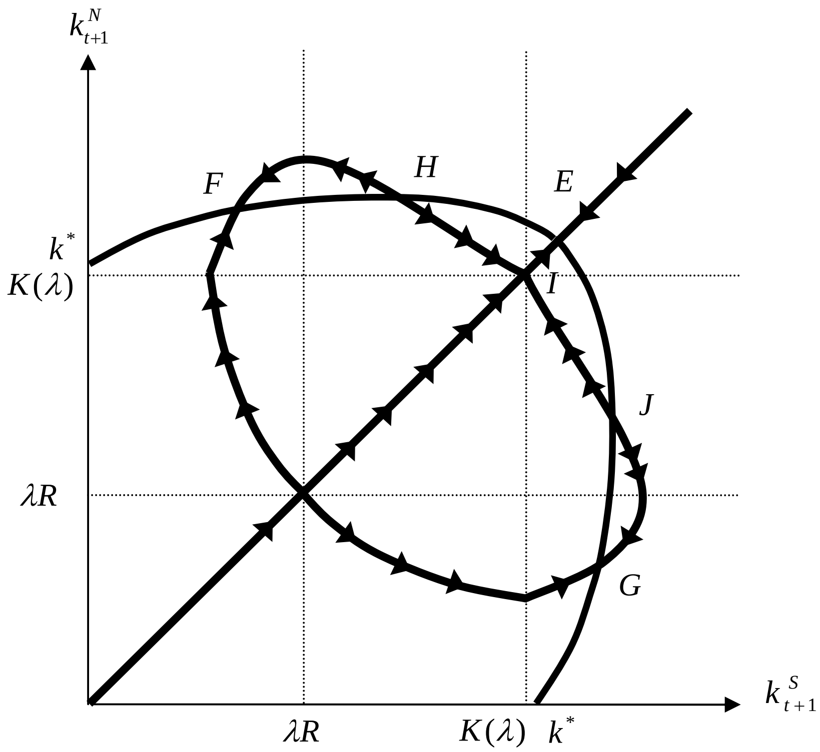

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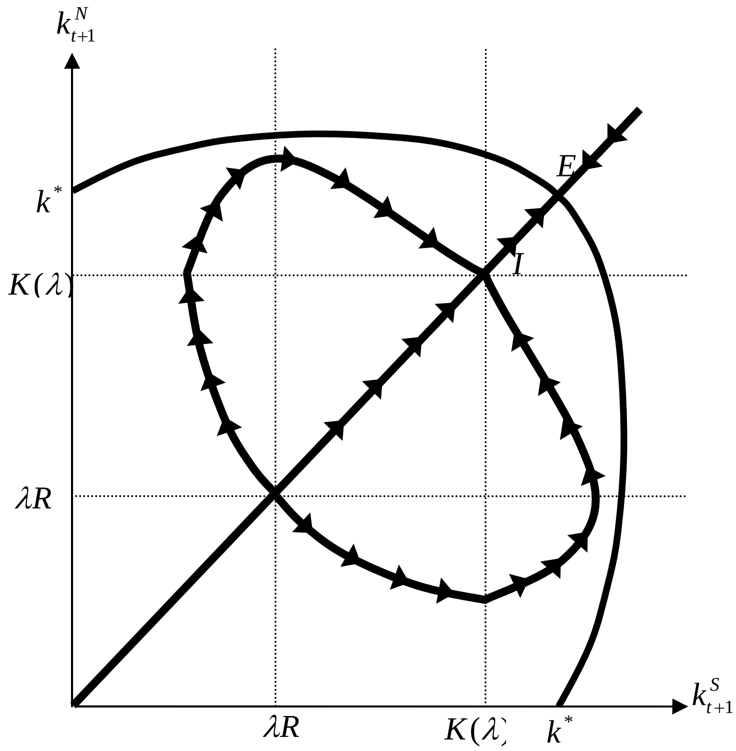

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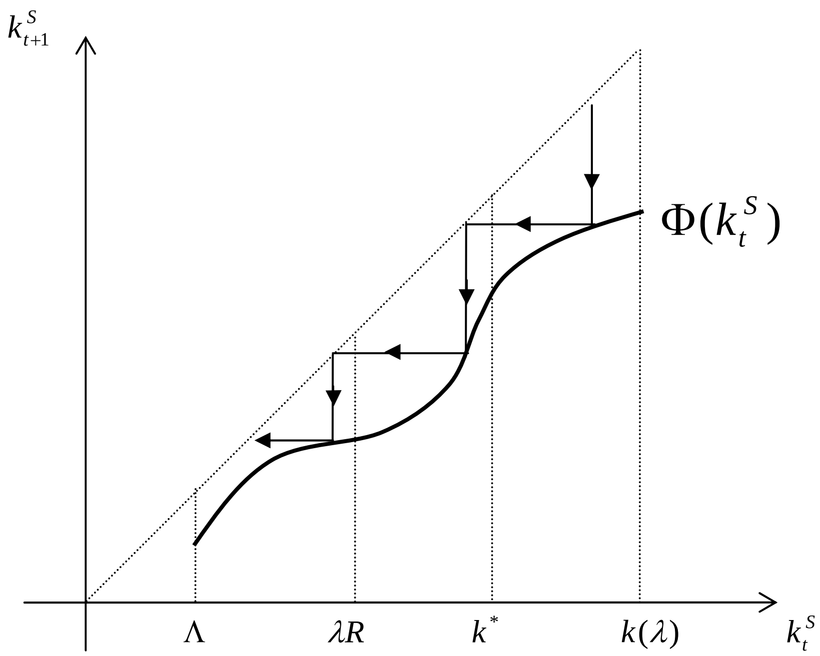

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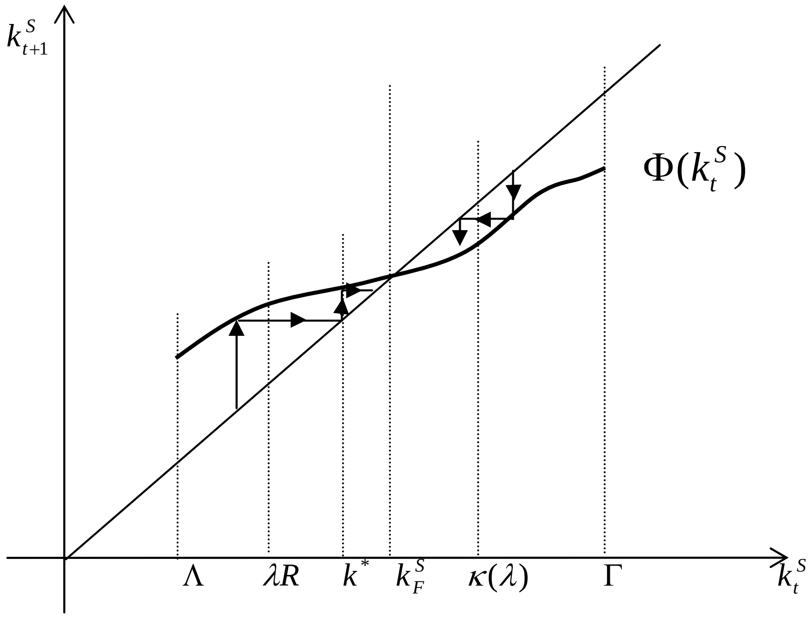

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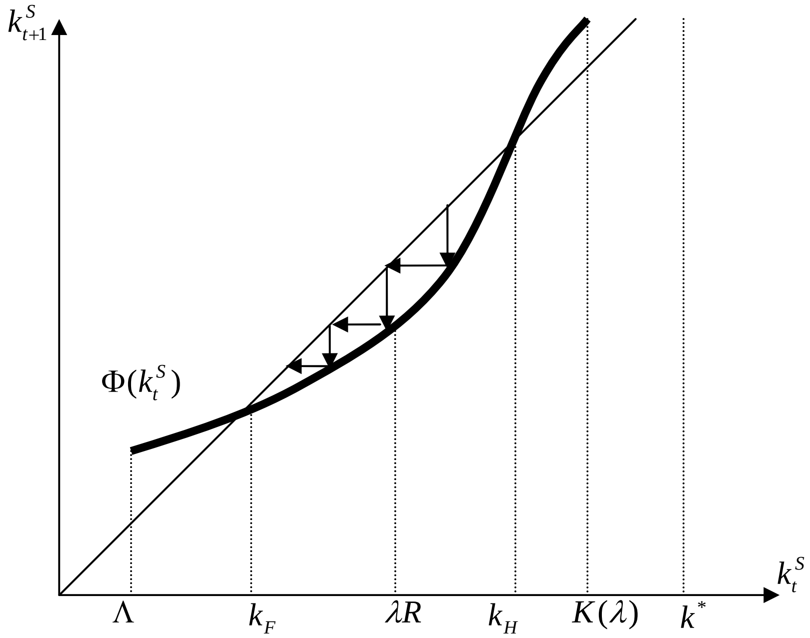

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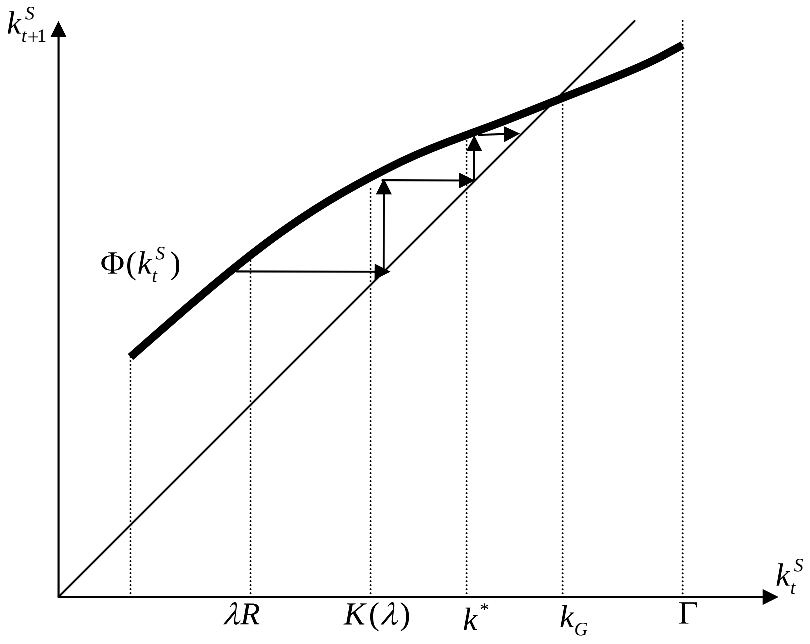

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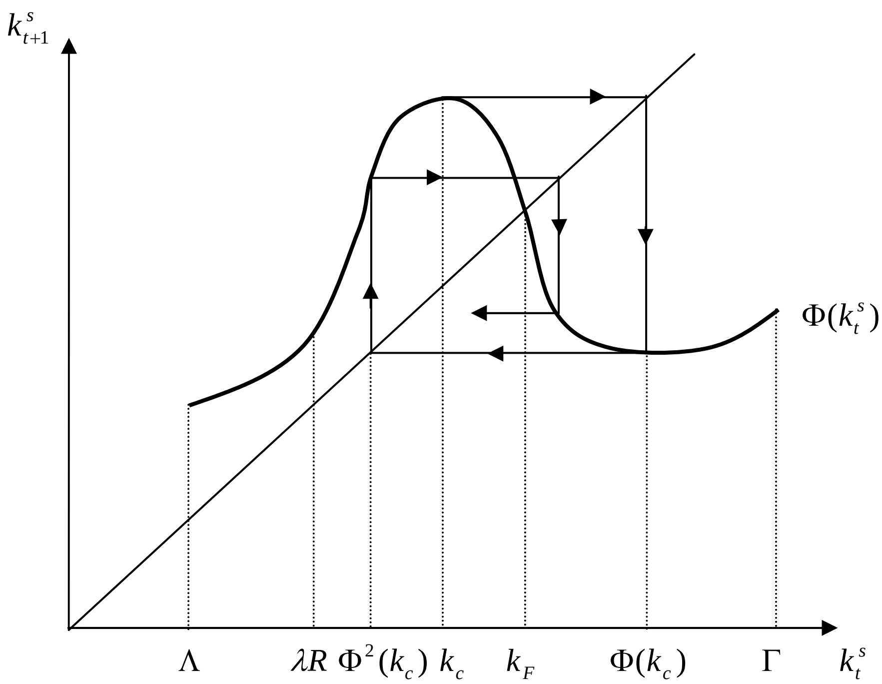
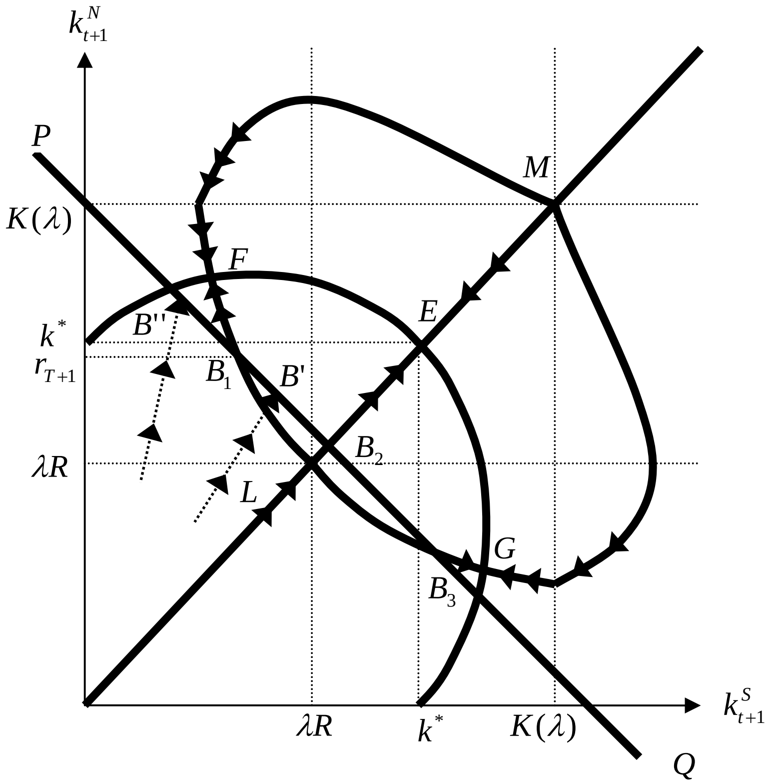


Figure 5-2A



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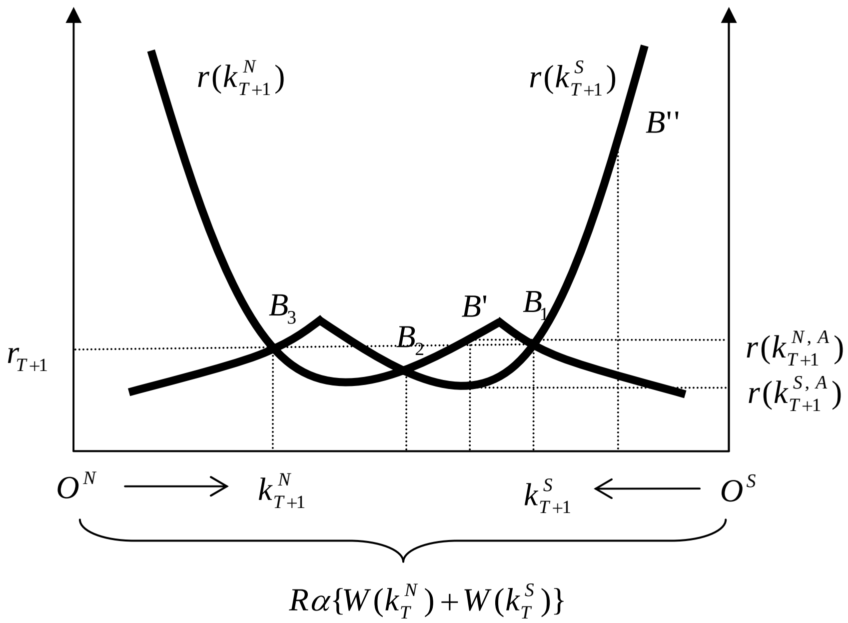


Figure 5-2C

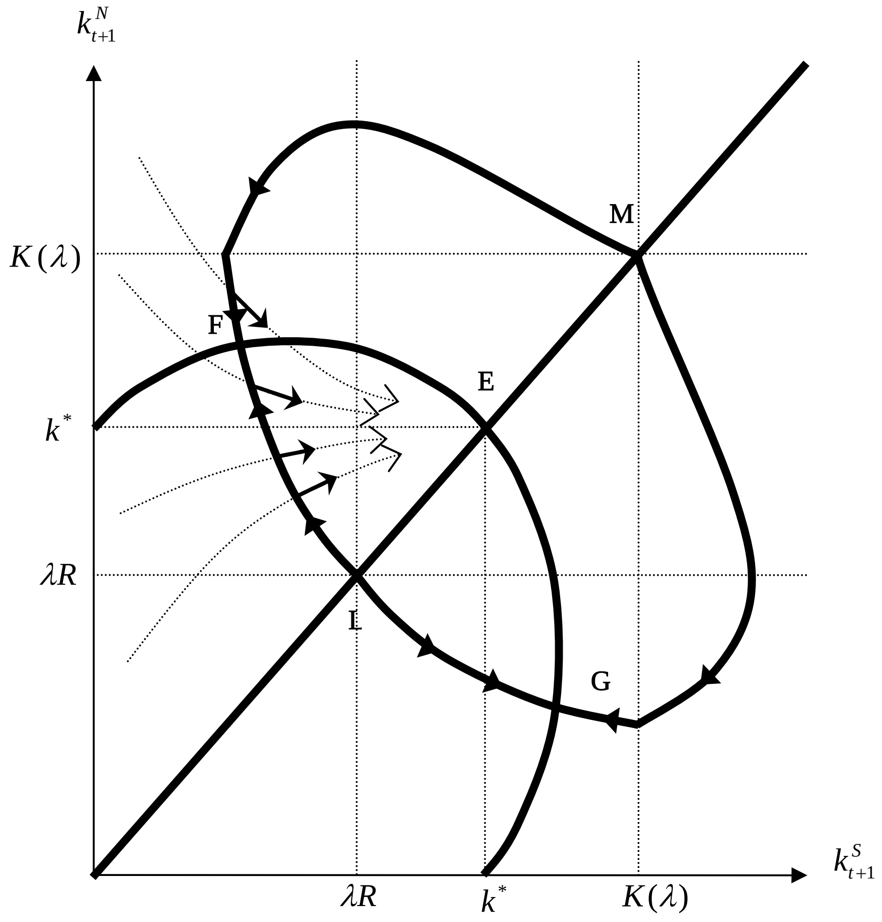


Figure 5-3A

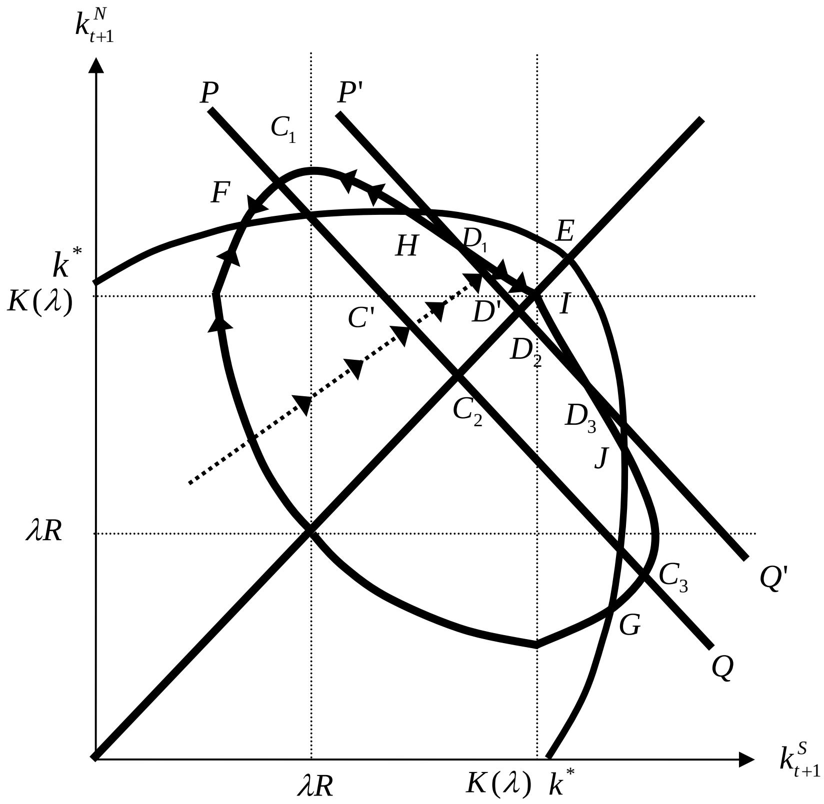


Figure 5-3B

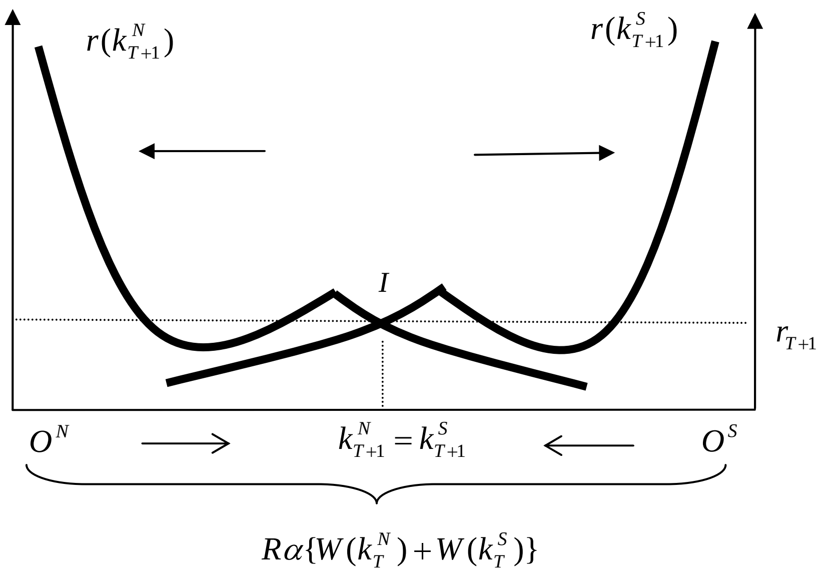


Figure 5-3C

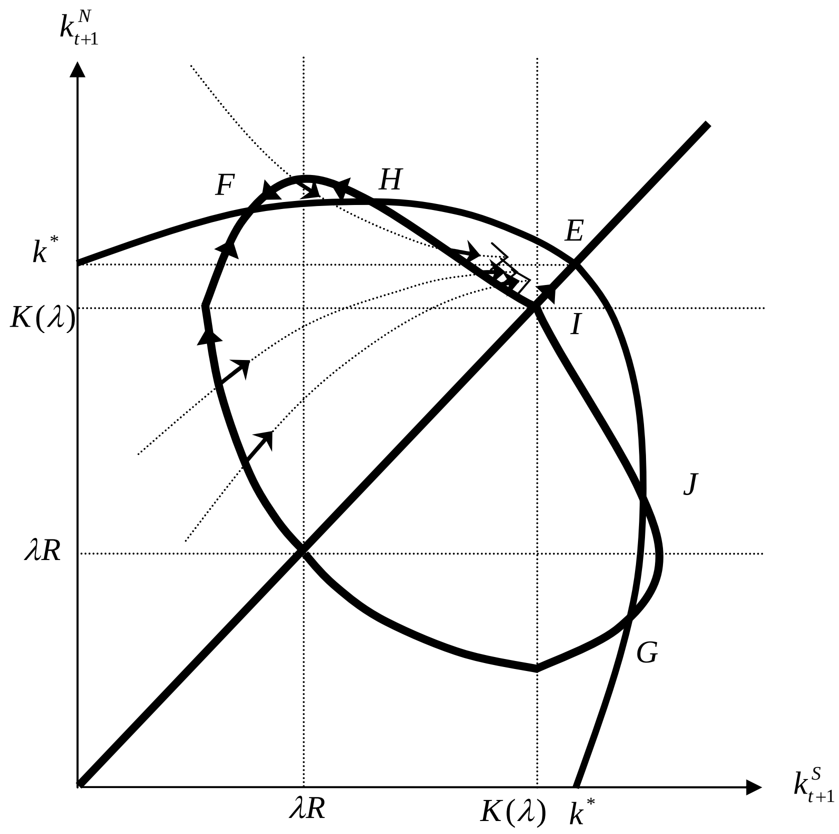


Figure5-3D

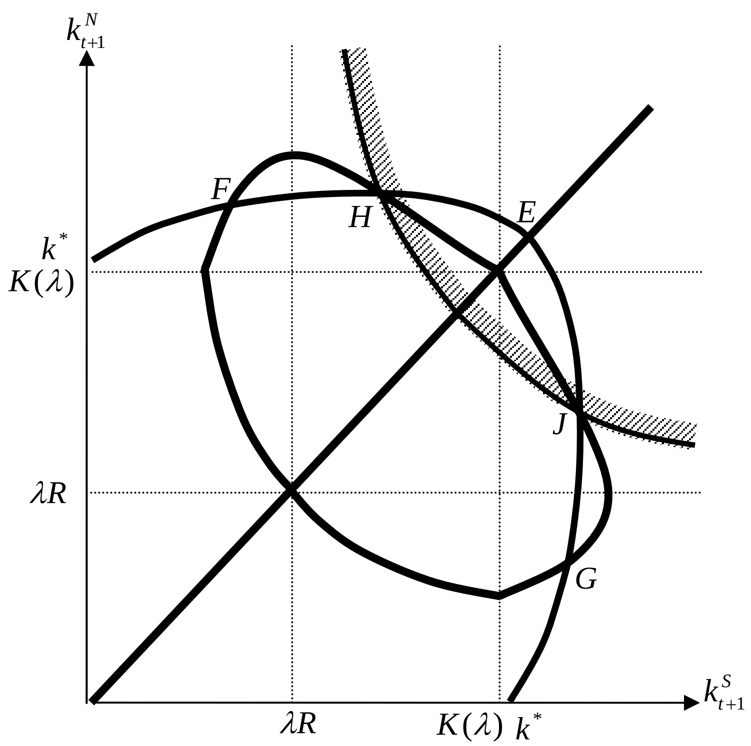


Figure5-4A

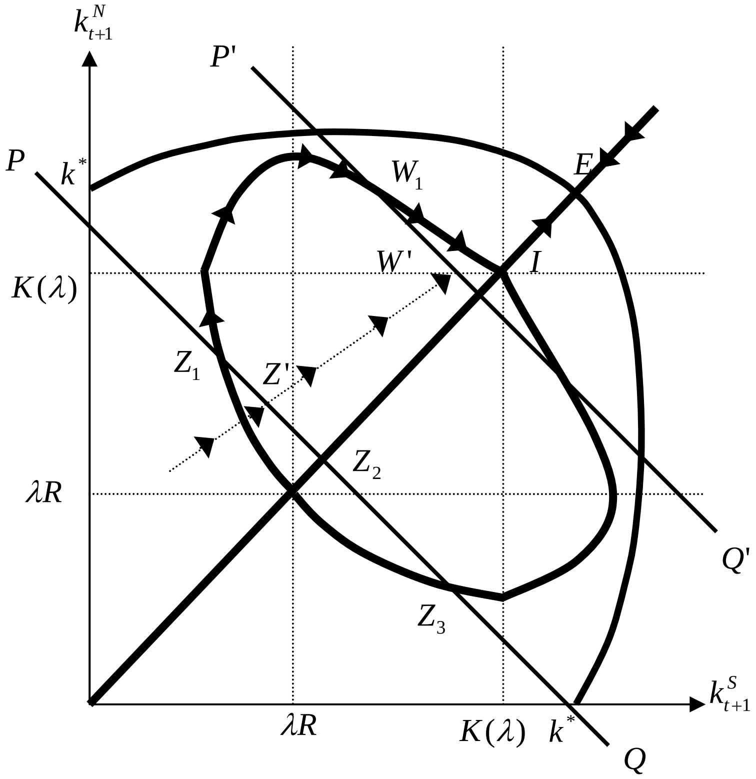


Figure5-4B

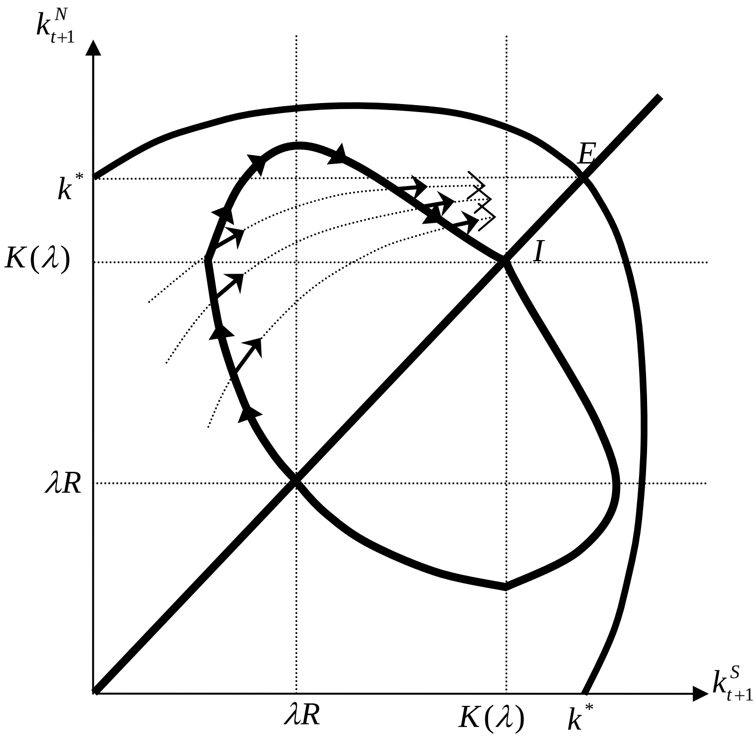
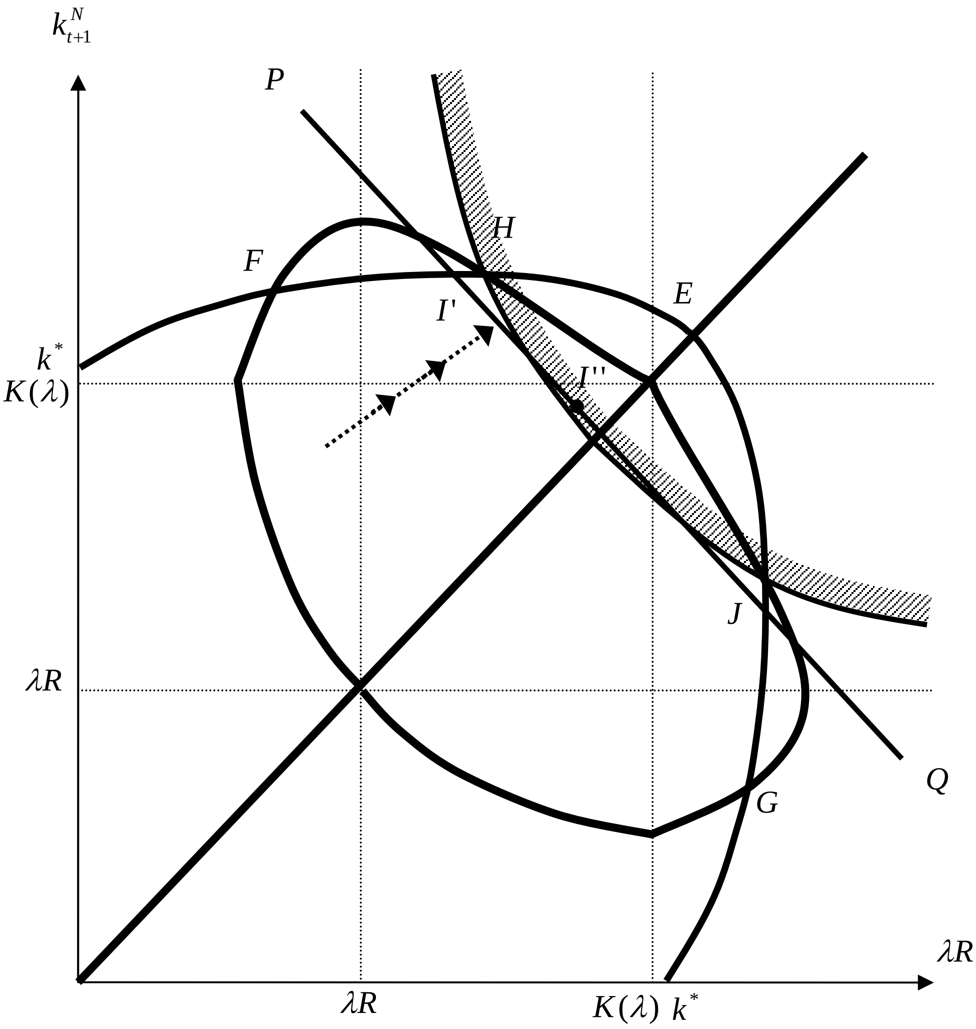


Figure6-1A


k_{t+1}^N
P
H
F
E
I'
I''
k^*
$K(\lambda)$
λR
J
Q
G
λR
λR
$K(\lambda)$ k^*