

**Land Price, Collateral, and Economic Growth <sup>#</sup>**

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**Abstract**

This paper extends Kiyotaki and Moore (1997, JPE) to an endogenous growth model and investigates dynamic properties of a growing economy with binding credit constraint when land is used not only as an input of production but also as collateral. There exists a balanced-growth path in an economy with binding credit constraint. In response to a once-and-for-all productivity shock, the developed model shows the propagation mechanism among output, capital, bank credit, and the land price in terms of the growth rate. The model's tractability allows us to derive interesting qualitative and quantitative findings.

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## 1. Introduction

It is widely recognized that land prices have played an important role in influencing credit market conditions, corporate investment, and output in Japan. Kiyotaki and Moore (1997, hereafter K&M) formulate a theoretical framework in which the appreciation in the collateral value mitigates credit constraints and thus amplifies fluctuations in investment and output when land serves as collateral for loans.<sup>1</sup> A number of empirical researches provide evidence to support the important role of the collateral channel, including Ogawa, Kitasaka, Yamaoka and Iwata (1996), Ogawa and Kitasaka (1998), and Ogawa and Suzuki (1998).

Over the post-war period, annual growth rates of GDP and land prices look like showing a positive correlation in Japan. Until the period of the asset price boom at the end of the 1980s, the persistent appreciation in land prices has been associated with faster economic growth. Meanwhile in the 1990s when one may call the “Lost Decade”, the persistent depreciation in land prices has been associated with the slowdown in economic growth. The average annual growth rates of GDP are 4.9% in the 1970s, 3.8% in the 1980s, and 1.5% in the 1990s, and the average growth rates of nationwide land prices are 9.2%, 6.1%, and -1.7%, respectively. Observing this time-series behavior motivates us to understand how the K&M transmission mechanism works in the growing economy.

In this paper we extend the K&M model to an endogenous growth model and investigate dynamic properties of a growing economy with binding credit constraint when land is used not only as an input of production but also as collateral. A balanced-growth path (BGP) is proved to exist when the credit constraint is binding when entrepreneurs who borrow money discount the future more than investors who lend to them. The equilibrium interest rate is determined to be smaller relative to an economy with no binding credit constraint. When borrowers discount

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<sup>1</sup> Since the seminal work by K&M, ample studies investigate the interaction between asset prices, credit, and output in the economy with binding credit constraint, including Edison *et al.* (2000) and Chen (2001) .

the future more and thus lenders discount the future less, the greater credit supply has to be adjusted by the smaller equilibrium interest rate than otherwise. The resulting discrepancy between the equilibrium interest rate and the marginal product of capital is, however, consistent with the equilibrium in the credit market because the borrowers' debt capacity is limited by the liquidation value of land. This aspect of the equilibrium is contrasted with other models of binding credit constraint, including K&M, Kiyotaki (1998), and Aghion *et al.* (1999), all of which identify the economy with binding credit constraint as the one with a smaller interest rate that is given exogenously.

In response to a once-and-for-all productivity shock, our model exhibits the propagation mechanism among output, capital, bank credit, and the land price in terms of the growth rate. This finding is contrasted with K&M that derive the propagation mechanism in terms of levels. The model's tractability allows us to characterize the whole process of business fluctuations. On impact, capital, the land price, and the equilibrium real interest rate show the procyclical behavior. Additionally, the model's tractability allows us to derive not only the qualitative but also the quantitative finding. The growth-enhancing effect of capital and output is shown to be greater in magnitude and is more persistent in the economy with binding credit constraint for plausible parameters.

By the introduction of a foreign asset into the model, the equilibrium with binding credit constraint is divided into two regimes, depending on parameters. In one regime, the equilibrium shows a BGP with the endogenous determination of the interest rate, while in another regime the interest rate is tied with the world interest rate and the equilibrium does not show a BGP. The former regime is more likely to emerge if either the productivity is high or the world interest rate is low.

This paper is organized as follows. In Section 2 we present the basic model. In Section 3 we study a benchmark economy with no borrowing constraint. In Section 4 we investigate an

economy with binding credit constraint. In Section 5 we study the impacts of a one-time productivity shock on the dynamic of the economy. In Section 6 we introduce the foreign asset with a constant interest rate into the model. Finally we conclude.

## 2. The Model

Consider an economy consisting of two types of agents who live infinitely, “entrepreneurs” and “investors”. The measures of entrepreneurs and investors are both unity. A significant difference between our model and K&M model is the production technology. We describe the production function as

$$Y_t = A_{t-1} K_{t-1}^\alpha L_{t-1}^{1-\alpha} \bar{K}_{t-1}^{1-\alpha}, \quad (1)$$

where  $Y_t$  is the output of production at period  $t$ ,  $K_{t-1}$  and  $L_{t-1}$  are capital stock and land which are used as inputs for production at period  $t-1$ ,  $\bar{K}_{t-1}$  represents the aggregate capital stock at period  $t-1$  that captures technological externalities (e.g. Romer, 1986), and  $A_{t-1}$  is the total factor productivity. Capital does not depreciate.

The preference of each of investors is described as  $\sum_{t=0}^{\infty} \beta^t \log C_t$ , where  $C_t$  is consumption at period  $t$ ,  $\beta \in (0,1)$  is the discount factor for future utilities, and  $E_t$  is the expectation formed at period  $t$ . At each period, there is a competitive one-period credit market, in which one unit of good at date  $t$  is exchanged for a claim to  $R_{t+1}$  units of good at period  $t+1$ .

On the other hand, the preference of each of entrepreneurs is described as

$$\sum_{t=0}^{\infty} (\theta\beta)^t \log C_t^E, \text{ where } C_t^E \text{ is the consumption of the entrepreneur at period } t, \text{ and } \theta \leq 1$$

is assumed. The case for  $\theta < 1$  captures a situation that entrepreneurs are more “myopic” than investors.

Each of entrepreneurs is initially endowed with  $K_0$  units of capital and one unit of land,

and each of investors is initially endowed with  $W_0$  units of the final good. The total amount of land is normalized unity. Only entrepreneurs have access to the production technology. Investors earn interest income by lending their wealth to entrepreneurs. We assume that  $\theta\beta(1 + \alpha A) > 1$ . As is obvious below, this assumption allows us to focus on a growing economy.

We make a critical assumption about financial arrangement. We assume that there is no enforcement mechanism to fulfill financial contracts between debtors and creditors. In this society, lenders cannot enforce on borrowers to repay their debt unless the debts are secured. In order to secure their debt, creditors can collect land that the debtor holds. Creditors cannot seize output or capital of their debtors. In this environment, anticipating the possibility of the borrower's strategic default, the creditor limits the amount of credit so that the debt repayment due at the next period will not exceed the value of land that the borrower possesses.

### 3. Benchmark Economy

We start with the analysis of a benchmark economy with no enforcement problem. In this frictionless economy BGP exists only when both entrepreneurs and investors discount futures at the same rate. Hence we focus on the case for  $\theta = 1$ .

Throughout this section and Section 4, we investigate an economy in the absence of uncertainty concerning the technology shock and assume that  $A_t = A$  for all  $t$ 's.

In the frictionless economy, each of entrepreneurs chooses a vector  $\{C_t^E, B_t, K_t, L_t\}_{t=0}^{\infty}$  to maximize  $\sum_{t=0}^{\infty} \beta^t \log C_t^E$  subject to the flow-of-fund constraint (FFC), given by

$$AK_{t-1}^{\alpha} L_{t-1}^{1-\alpha} \bar{K}_{t-1}^{1-\alpha} + B_t - R_t B_{t-1} \geq K_t - K_{t-1} + Q_t(L_t - L_{t-1}) + C_t^E, \quad (2)$$

given  $K_0 > 0$  and  $L_0 = 1$ . Note that  $B_t$  is the amount of borrowing,  $Q_t$  is the land price at period  $t$  and  $R_t$  is the interest rate that is determined at  $t-1$  and is promised to repay at  $t$ . The sequence of events is described Figure 1.

Each of investors chooses a vector  $\{C_t, W_t\}_{t=0}^{\infty}$  to maximize  $\sum_{t=0}^{\infty} \beta^t \log C_t$  subject to the budget constraint, given by

$$W_t = R_t W_{t-1} - C_t, \quad (3)$$

given  $W_0 > 0$ , where  $W_t$  is the asset that the investor holds at the beginning of period  $t$ . We obtain  $R_t = 1 + \alpha A$ , using  $K_t = \bar{K}_t$  and  $L_t = 1$  on the ground that entrepreneurs are homogeneous and that entrepreneurs only demand land. Investors earn interest income by lending to entrepreneurs at the interest rate  $\alpha A$ . The first-order condition for land implies

$$(1 - \alpha)AK_t + Q_{t+1} = (1 + \alpha A)Q_t, \quad (4)$$

where we again use  $K_t = \bar{K}_t$  and  $L_t = 1$ .

We characterize a BGP in the frictionless economy by

$$\frac{K_{t+1}}{K_t} = \frac{W_{t+1}}{W_t} = \beta(1 + \alpha A), \quad (5)$$

given the two transversality conditions,  $\lim_{t \rightarrow \infty} \beta^t (W_t / C_t) = 0$  and  $\lim_{t \rightarrow \infty} \beta^t (K_t / C_t) = 0$ .

Solving (4) forward, using (5) and the no-bubble condition for the land price,

$\lim_{t \rightarrow \infty} (1 + \alpha A)^{-t} Q_t = 0$ , we finally obtain

$$Q_t = \frac{(1 - \alpha)A}{(1 + \alpha A)(1 - \beta)} K_t. \quad (6)$$

Equations (5) and (6) jointly imply that there exists a BGP with a growth rate of  $\beta(1 + \alpha A)$  on the wealth of investors  $W_t$ , capital invested by entrepreneurs  $K_t$ , and the land price.

#### 4. Economy with binding Credit Constraint

Now we turn to the analysis of an economy in which there is potentially an enforcement problem. As will be made clear, a BGP will exist also when  $\theta < 1$ . Each of entrepreneurs

chooses a vector  $\{C_t^E, B_t, K_t, L_t\}_{t=0}^{\infty}$  to maximize  $\sum_{t=0}^{\infty} (\theta\beta)^t \log C_t^E$  subject to the FFC given

by (2), and the borrowing constraint (BC) given by

$$R_{t+1}B_t \leq Q_{t,t+1}^e L_t, \quad (7)$$

given  $K_0 > 0, B_0 > 0$ , and  $L_0 = 1$ , where  $Q_{t,t+1}^e$  is the expected land price at period  $t + 1$  in which the expectation is formed at period  $t$ . Equation (7) states that entrepreneurs can borrow the amount so that the debt repayment that is obliged to make at  $t + 1$  will not exceed the *expected* liquidation value of the land as of  $t$ .

Solving for the entrepreneur's problem, we formulate the following Lagrangean function as

$$\begin{aligned} \Lambda(C_t^e, K_t, L_t, B_t) = & \sum_{t=0}^{\infty} (\theta\beta)^t [\log C_t^E \\ & + \lambda_t \{ AK_{t-1}^\alpha L_{t-1}^{1-\alpha} \bar{K}_{t-1}^{1-\alpha} + B_t - R_t B_{t-1} - (K_t - K_{t-1}) - Q_t(L_t - L_{t-1}) - C_t^E \} \\ & + \eta_t \{ Q_{t,t+1}^e L_t - R_{t+1} B_t \}], \end{aligned}$$

where  $\lambda_t$  and  $\eta_t$  are the Lagrange multipliers. Optimal conditions are given by four

first-order conditions,  $\frac{\partial \Lambda}{\partial C_t^E} = 0$ ,  $\frac{\partial \Lambda}{\partial K_t} = 0$ ,  $\frac{\partial \Lambda}{\partial L_t} = 0$ , and  $\frac{\partial \Lambda}{\partial B_t} = 0$ , two

complementary slacknesses,  $\lambda_t \geq 0$ ,  $\frac{\partial \Lambda}{\partial \lambda_t} \geq 0$  or  $\lambda_t \times \frac{\partial \Lambda}{\partial \lambda_t} = 0$ , and  $\eta_t \geq 0$ ,

$\frac{\partial \Lambda}{\partial \eta_t} \geq 0$  or  $\eta_t \times \frac{\partial \Lambda}{\partial \eta_t} = 0$ , and the transversality condition,  $\lim_{T \rightarrow \infty} \beta^T \lambda_T K_T = 0$ .

When  $K_t = \bar{K}_t$  and  $L_t = 1$  are used, first-order conditions are described as

$$\lambda_t = (C_t^E)^{-1}, \quad (8)$$

$$\frac{\lambda_t}{\lambda_{t+1}} = \theta\beta(1 + \alpha A), \quad (9)$$

$$\frac{\lambda_t}{\lambda_{t+1}} Q_t = \frac{\eta_t}{\lambda_{t+1}} Q_{t+1}^e + \theta\beta\{(1 - \alpha)AK_t + Q_{t+1}\}, \quad (10)$$

and

$$\frac{\lambda_t}{\lambda_{t+1}} = \left( \frac{\eta_t}{\lambda_{t+1}} + \theta\beta \right) R_{t+1}. \quad (11)$$

The market clearing in the credit market requires the aggregate demand for credit to be equal to the aggregate supply of funds, and is given by

$$B_t = W_t. \quad (12)$$

Finally, the investor's optimal behavior remains to satisfy

$$\frac{W_{t+1}}{W_t} = \beta R_{t+1}. \quad (13)$$

Now we are prepared to solve the equilibrium. It follows from (8) and (9) that

$$\frac{C_{t+1}^E}{C_t^E} = \theta\beta(1 + \alpha A). \quad (14)$$

It follows from (9) and (11) that

$$\theta\beta(1 + \alpha A) = \left( \frac{\eta_t}{\lambda_{t+1}} + \theta\beta \right) R_{t+1}. \quad (15)$$

Equation (15) characterizes the condition under which the economy is (or is not) credit-constrained. It is obvious from (8) that  $\lambda_t > 0$  for all  $t \geq 0$ , and thus (15) implies that  $R_t < 1 + \alpha A$  holds if and only if  $\eta_t > 0$ . Two cases are to be distinguished: in one case  $R_t < 1 + \alpha A$  holds and the BC given by (7) is binding with equality, and in the other case  $R_t = 1 + \alpha A$  holds and the BC given by (7) is not binding with equality.

Consider first the case when the BC given by (7) is binding with equality. Under the assumption of perfect foresight,  $Q_{t,t+1}^e = Q_{t+1}$  holds. Together with this, it follows from (9), (10), and (11) that

$$(1 + \alpha A)Q_t = (1 - \alpha)AK_t + \frac{1 + \alpha A}{R_{t+1}}Q_{t+1}. \quad (16)$$

Combining the FFC given by (2) with the BC given by (7), together with  $K_t = \bar{K}_t$  and

$L_t = 1$ , leads to

$$C_t^E = (1 + A)K_{t-1} - K_t + R_{t+1}^{-1}Q_{t+1} - Q_t. \quad (17)$$

The entrepreneur's consumption is rewritten, using (16), as

$$C_t^E = (1 + A)K_{t-1} - \frac{1 + A}{1 + \alpha A}K_t. \quad (18)$$

It follows from (14) and (18) that



$$\frac{(1+A)K_t - \frac{1+A}{1+\alpha A} K_{t+1}}{(1+A)K_{t-1} - \frac{1+A}{1+\alpha A} K_t} = \theta\beta(1+\alpha A). \quad (19)$$

Letting  $g_t^K \equiv K_t/K_{t-1}$  be the growth rate of capital, rearrangement of (19) leads to

$$\theta\beta(1+\alpha A)(1+A)(g_t^K)^{-1} = (1+\theta\beta)(1+A) - \frac{1+A}{1+\alpha A} g_{t+1}^K. \quad (20)$$

Further rearrangement of (20) leads to

$$g_{t+1}^K = (1+\alpha A)(1+\theta\beta) - \theta\beta(1+\alpha A)^2 (g_t^K)^{-1} \equiv F(g_t^K). \quad (21)$$

The Function  $F(g_t^K)$  is increasing, bounded above from  $(1+\alpha A)(1+\theta\beta)$ , and satisfies

$\lim_{g_t^K \rightarrow 0} F(g_t^K) = -\infty$ . Letting  $g^K \equiv g_t^K = g_{t+1}^K$  be the stationary growth rate of capital, there

are two solutions,  $\theta\beta(1+\alpha A)$  and  $1+\alpha A$ . Figure 2 illustrates the upward sloping curve  $F(g_t^K)$  that intersects the 45-degree line at two points. Since the higher stationary growth rate  $1+\alpha A$  violates the transversality condition, the possible equilibrium is only around the smaller stationary growth rate  $\theta\beta(1+\alpha A)$ . Since the stationary state is dynamically unstable, all the exploding paths violate the transversality condition. Hence the stationary growth path with a growth rate of

$$g^K = \theta\beta(1+\alpha A) \quad (22)$$

is the unique equilibrium. We now turn to the determination of the interest rate. It follows from

(12), (13), and the BC given by (7) that  $\beta R_{t+1} = \frac{B_{t+1}}{B_t} = \frac{Q_{t+2}/R_{t+2}}{Q_{t+1}/R_{t+1}}$ , and thus

$$\beta R_{t+1} = \frac{Q_{t+1}}{Q_t}. \quad (23)$$

It follows from (16) and (23) that

$$\frac{K_t}{Q_t} = \frac{(1+\alpha A)(1-\beta)}{(1-\alpha)A}. \quad (24)$$

Equation (24) implies that capital and the land price grow at the same rate. It follows from (22) and this aspect of the BGP that

$$\frac{K_{t+1}}{K_t} = \frac{Q_{t+1}}{Q_t} = \theta\beta(1+\alpha A). \quad (25)$$

Combining (25) with (23) leads to the interest rate:

$$R_t = \theta(1+\alpha A). \quad (26)$$

Notice finally that  $R_t < 1 + \alpha A$ , which requires  $\theta < 1$ . There exists an economy with binding credit constraint only when entrepreneurs are more myopic than investors. Therefore, if  $\theta < 1$ , the equilibrium is credit-constrained, and exhibits a BGP, where capital, the land price, and the wealth of investors grow at the rate  $\theta\beta(1 + \alpha A)$ .

Consider secondly the case when the BC given by (7) is not binding with equality. It follows from (9) and (10) that

$$(1 + \alpha A)Q_t = (1 - \alpha)AK_t + Q_{t+1}. \quad (27)$$

The relation  $R_{t+1} = 1 + \alpha A_t$  follows from (15) with  $\eta_t = 0$ . Combining the FFC given by (2) with (27) and the above relation leads to

$$(1 + \alpha A)(K_{t-1} + Q_{t-1} - B_{t-1}) - (K_t + Q_t - B_t) = C_t^E. \quad (28)$$

Letting  $E_t \equiv K_t + Q_t - B_t$  be the firm's equity value, it follows from (14) and (26) that

$$\frac{(1 + \alpha A)E_t - E_{t+1}}{(1 + \alpha A)E_{t-1} - E_t} = \theta\beta(1 + \alpha A). \quad (29)$$

From the analogous analysis as the economy with binding credit constraint, the unique equilibrium should satisfy

$$\frac{E_{t+1}}{E_t} = \theta\beta(1 + \alpha A). \quad (30)$$

Equation (30) implies that the firm's equity value grows at the rate  $\theta\beta(1 + \alpha A)$ , while the wealth of investors grows at the rate  $\beta(1 + \alpha A)$ , which is implied by (13) and  $R_t = 1 + \alpha A$ . Considering a BGP in which  $K_t$ ,  $Q_t$ , and  $W_t$  grow at the same rate, the equilibrium is feasible only when  $\theta = 1$ . Therefore, if  $\theta = 1$ , the equilibrium is not credit-constrained, and exhibits a BGP, where capital, the land price, and the wealth of investors grow at the rate  $\beta(1 + \alpha A)$ . The equilibrium is summarized in the following proposition.

**Proposition 1:**

*(Case A) Assume that  $\theta = 1$ . The economy is not credit-constrained. The equilibrium exhibits a BGP with a growth rate  $\beta(1 + \alpha A)$  and the interest rate  $1 + \alpha A$ .*

*(Case B) Assume that  $\theta < 1$ . The economy is credit-constrained. The equilibrium exhibits a BGP with a growth rate  $\theta\beta(1 + \alpha A)$  and the interest rate  $\theta(1 + \alpha A)$ .*

There exists a BGP in an economy with binding credit constraint when the equilibrium interest rate is endogenously determined to clear the credit market. In other words, the endogenous

behavior of the interest rate allows the economy to grow on the BGP. A simple experiment makes this point transparent. Assume that a permanent productivity shock arrives. A permanent rise in the productivity  $A$  leads to a rise in the land price, and allows borrowers to demand more credit because the appreciation in land mitigates credit constraints faced by them. This, in turn, drives the interest rate up, and allows investors to supply credit at higher speed. The rise in  $A$  makes the resource allocation with higher growth rates of capital, the land price, and wealth of entrepreneurs and investors sustainable. This theoretical finding is contrasted with K&M, Kiyotaki [1998] and Aghion *et al.* [1999], all of which identify the economy with binding credit constraint as the one with an exogenously given interest rate that is smaller than the return of productive asset.

When borrowers discount the future more and thus lenders discount the future less, the greater credit supply has to be adjusted by the smaller equilibrium interest rate than otherwise. The resulting discrepancy between the equilibrium interest rate and the marginal product of capital is, however, consistent with the equilibrium in the credit market because the borrowers' debt capacity is limited by the liquidation value of land.

## 5. Once-And-For-All Productivity Shock and Business Fluctuations

Having characterized the equilibrium of the economy with binding credit constraint, we are prepared to explore a number of implications of business cycles. We consider the response of an unanticipated once-and-for-all productivity shock on the dynamics of the economy. We assume that at the beginning of period  $T$  people experience a once-and-for-all productivity shock from  $A$  to  $A'$  with  $A' > A$ .

We first examine the effect of the shock in the economy with no binding credit constraint. We denote time subscript for  $A$  as necessity. For reference, it will be useful to express the counterpart of (21) as the one with time subscript for  $A$ , i.e.,

$$g_{t+1}^K = (1 + \theta\beta) \frac{(1 + \alpha A_{t+1})(1 + A_t)}{1 + A_{t+1}} - \theta\beta \frac{(1 + \alpha A_{t+1})(1 + \alpha A_t)(1 + A_{t-1})}{(1 + A_{t+1})g_t^K}. \quad (21')$$

Letting  $g_t^E \equiv \frac{E_t}{E_{t-1}}$  be the growth rate of the firm's equity value, (29) is rewritten, with

$\theta = 1$ , as

$$\beta(1 + \alpha A_{t-1})(1 + \alpha A_t)(g_t^E)^{-1} - (1 + \beta)(1 + \alpha A_t) + g_{t+1}^E = 0. \quad (31)$$

Assuming that  $A_t = A' > A = A_{T-1} = A_{T+1} = A_{T+2} = \dots$ , it is straightforward to find

$g_{T+2}^E = \beta(1 + \alpha A)$ . Introducing  $g_{T+2}^E = \beta(1 + \alpha A)$  into (31) with  $t = T + 1$  leads to

$g_{T+1}^E = \beta(1 + \alpha A')$ . Introducing further  $g_{T+1}^E = \beta(1 + \alpha A')$  into (31) with  $t = T$  leads to

$$g_T^E = \beta(1 + \alpha A).$$

It follows from (12), (13) and  $R_{t+1} = 1 + \alpha A_t$  that  $\frac{B_{t+1}}{B_t} = \frac{W_{t+1}}{W_t} = \beta(1 + \alpha A_t)$ . Since

$E_t + B_t = K_t + Q_t$  holds by definition, it follows that

$$\frac{K_{t+1} + Q_{t+1}}{K_t + Q_t} = \beta(1 + \alpha A_t). \quad (32)$$

Letting  $g_{t+1}^Q \equiv \frac{Q_{t+1}}{Q_t}$  be the growth rate of the land price, (27) is rewritten as

$$\frac{K_t}{Q_t} = \frac{1 + \alpha A_t - g_{t+1}^Q}{(1 - \alpha)A_t}. \quad (33)$$

It follows from (32) and (33) that

$$g_{t+1}^Q = \frac{\beta(1 + A_t)(1 + \alpha A_t)A_{t+1}}{\beta(1 + \alpha A_t)A_{t+1} + (1 + A_{t+1} - g_{t+2}^Q)A_t}. \quad (34)$$

Since it is straightforward to see  $g_{T+2}^Q = \beta(1 + \alpha A)$ , the growth rate of the land price at period  $T + 1$  is finally expressed as

$$g_{T+1}^Q = \frac{\beta(1 + A')(1 + \alpha A')A}{\beta(A - A') + A'(1 + A)}. \quad (35)$$

We calculate  $g_T^Q$  by plugging the predetermined value of  $\frac{K_{T-1}}{Q_{T-1}} = \frac{(1 - \beta)(1 + \alpha A)}{(1 - \alpha)A}$ , given by

(6), into (33). We finally obtain  $g_T^Q = \beta(1 + \alpha A)$ .

We next turn to the determination of  $g_t^K$ . Equation (27) is rewritten as

$$\frac{g_{t+1}^K + g_{t+1}^Q(K_t / Q_t)^{-1}}{1 + (K_t / Q_t)^{-1}} = \beta(1 + \alpha A_t). \quad (36)$$

It follows from (33) and (36) that  $g_t^K$  becomes a function of  $g_t^Q$ , such that

$$g_t^K = \frac{\beta(1 + \alpha A_{t-1})(1 + A_{t-1}) - \{\beta(1 + \alpha A_{t-1}) + (1 - \alpha)A_{t-1}\}g_t^Q}{1 + \alpha A_{t-1} - g_t^Q}. \quad (37)$$

Substituting (35) into (37) leads to

$$g_{T+1}^K = \beta(1 + \alpha A) \frac{(1 + A')}{(1 + A)}. \quad (38)$$

We obtain the growth rate of capital at  $T$  as  $g_T^K = \beta(1 + \alpha A)$  by plugging

$$g_T^Q = \beta(1 + \alpha A) \quad \text{and} \quad \frac{K_{T-1}}{Q_{T-1}} = \frac{(1 - \beta)(1 + \alpha A)}{(1 - \alpha)A} \quad \text{into (36). Using the relation } R_t = 1 + \alpha A,$$

we obtain interest rates as  $R_{T+2} = 1 + \alpha A$ ,  $R_{T+1} = 1 + \alpha A'$ , and  $R_T = 1 + \alpha A$ . Note that

none of variables at  $T$ ,  $K_T$ ,  $Q_T$ , or  $R_T$ , reacts to any contemporaneous change in  $A_T$ .

Entrepreneurs who have the log-utility do not change their consumption/investment decision to the change in the interest rate. The impact of the shock arises with a one-period lag.<sup>2</sup>

We turn to the analysis of the economy with binding credit constraint. In response to the productivity shock, under the assumption of perfect foresight  $Q_{t,t+1}^e = Q_{t+1}$  holds for any  $t \geq T + 1$ , and the consumption of entrepreneur is described as (17) for  $t \geq T + 1$ . However, at  $T$  when the shock arrives,  $Q_{T-1,T}^e = Q_T$  will be in general violated. The entrepreneur's consumption at period  $T$  can be expressed as

$$\begin{aligned} C_T^E &= (1 + A_{T-1})K_{T-1} - K_T + R_{T+1}^{-1}Q_{T+1} - Q_T + Q_T - Q_{T-1,T}^e \\ &= (1 + A_{T-1})K_{T-1} - \frac{1 + A_T}{1 + \alpha A_T} K_T + (Q_T - Q_{T-1,T}^e). \end{aligned} \quad (39)$$

Notice that the last term  $(Q_T - Q_{T-1,T}^e)$  captures the gain (lose) from the unexpected change in the land price. Since it is straightforward to find  $g_{T+2}^K = \theta\beta(1 + \alpha A)$ , it follows from (21) that

$$g_{T+1}^K = \theta\beta(1 + \alpha A) \frac{1 + A'}{1 + A}, \quad (40)$$

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<sup>2</sup> Both  $K_T$  and  $Q_T$  will react to the shock to the extent that entrepreneurs have an incentive to change their consumption/investment decision to the shock. If, instead, the preference of entrepreneurs is described as  $\sum_{t=0}^{\infty} (\theta\beta)^t \frac{1}{1 - \sigma} (C_t^E)^{1 - \sigma}$  ( $\sigma \neq 1$ ),  $K_T$  and  $Q_T$  react to the shock.

which is qualitatively the same as the economy with no binding credit constraint. It follows from (14), (17), and (39) that

$$\frac{(1 + A_T) - \frac{1 + A_{T+1}}{1 + \alpha A_{T+1}} g_{T+1}^K}{(1 + A_{T-1})(g_T^K)^{-1} - \frac{1 + A_T}{1 + \alpha A_T} + \frac{Q_T - Q_{T-1,T}^e}{K_T}} = \theta\beta(1 + \alpha A_T). \quad (41)$$

Plugging (40) into (41) leads to

$$g_T^K = \frac{\theta\beta(1 + \alpha A_T)(1 + A_{T-1})}{(1 + A_T) - \theta\beta(1 + \alpha A_T)\Delta q_T}, \quad (42)$$

where  $\Delta q_T \equiv \frac{Q_T - Q_{T-1,T}^e}{K_T}$ . Notice that  $\Delta q_T > 0$  corresponds to the case when the debt

repayment is contingent on the *anticipated* land price, while  $\Delta q_T = 0$  will correspond to the case when the debt repayment is contingent on the *realized* land price.<sup>3</sup>

It will be useful to examine first the case for  $\Delta q_T = 0$ . Equation (42) then reduces to

$$g_T^K = \theta\beta \frac{(1 + \alpha A')(1 + A)}{(1 + A')},$$

which should be smaller than the growth rate of capital unless the productivity shock would arrive, i.e.,  $\beta(1 + \alpha A)$ . In response to the increase in  $Q_T$ , the debt repayment due at period  $T$  would rise, and hence entrepreneurs who find their smaller net worth would have to shrink investment in capital.

Consider next the interesting case for  $\Delta q_T > 0$ , which we should study in details. It

follows from (24) and (25) that  $\frac{Q_T}{K_T} = \frac{(1 - \alpha)A'}{(1 + \alpha A')(1 - \beta)}$  and

$$\frac{Q_{T-1,T}^e}{K_{T-1}} = \frac{Q_{T-1,T}^e}{Q_{T-1}} \frac{Q_{T-1}}{K_{T-1}} = \frac{\theta\beta(1 - \alpha)A}{1 - \beta}. \quad \text{Note that the latter comes from the fact that } \frac{Q_{T-1,T}^e}{Q_{T-1}} \text{ is}$$

equal to the value unless the shock would arrive. Finally,  $\Delta q_T$  is given by

$$\Delta q_T = \frac{Q_T}{K_T} - \frac{Q_{T-1,T}^e}{K_{T-1}} \frac{K_{T-1}}{K_T} = \frac{(1 - \alpha)A'}{(1 + \alpha A')(1 - \beta)} - \frac{\theta\beta(1 - \alpha)A}{1 - \beta} (g_T^K)^{-1}. \quad (43)$$

Plugging (43) into (42), and rearranging terms, the growth rate of capital at period  $T$

<sup>3</sup> This case might be viable if the timing of debt repayment is after the arrival of the productivity shock.

becomes

$$g_T^K = \theta\beta(1 + \alpha A') \frac{(1 + A) - \frac{\theta\beta(1 - \alpha)A}{1 - \beta}}{(1 + A') - \frac{\theta\beta(1 - \alpha)A'}{1 - \beta}}. \quad (44)$$

The following is established.

**Proposition 2:** *Assume that at the beginning of period  $T$  there is an unanticipated once-and-for-all productivity shock. The growth rate of capital at period  $T$  is higher in the economy with binding credit constraint than without it if  $\theta\beta + \beta > 1$ , and the growth rate of capital at period  $T + 1$  is smaller in the economy with binding credit constraint than without it.*

*Proof:* We prove the former part by comparing  $g_T^K$  in (44) and  $\theta\beta(1 + \alpha A)$ . Defining

$$\phi(A) \equiv \frac{1 + \alpha A}{1 + A - \{\theta\beta(1 - \alpha)A\}/(1 - \beta)}, \quad \phi(\cdot) \text{ is increasing if and only if } \theta\beta + \beta > 1.$$

Together with this fact, the comparison leads to the former part. The latter part is straightforward from the comparison between (38) and (40). Q.E.D.

Note that  $\theta\beta + \beta > 1$  is likely to be met since both  $\beta$  and  $\theta$  are deemed close to unity.

We now turn to the determination of the growth rates of the land price. It is useful to express

the counterpart of (24) as  $\frac{Q_t}{K_t} = \frac{(1 - \alpha)A_t}{(1 + \alpha A_t)(1 - \beta)}$ . We obtain  $\frac{Q_T}{K_T} = \frac{(1 - \alpha)A'}{(1 + \alpha A')(1 - \beta)}$ , and

$\frac{Q_t}{K_t} = \frac{(1 - \alpha)A}{(1 + \alpha A)(1 - \beta)}$  for any  $t$  except for  $t = T$ . It is straightforward to see

$g_{T+2}^Q = \beta(1 + \alpha A)$ . Further calculation leads to

$$g_{T+1}^Q = \frac{Q_{T+1}}{K_{T+1}} g_{T+1}^K \frac{K_T}{Q_T} = \theta\beta(1 + \alpha A') \frac{(1 + A')}{(1 + A)} \frac{A}{A'}, \quad (45)$$

and

$$g_T^Q = \frac{Q_T}{K_T} g_T^K \frac{K_{T-1}}{Q_{T-1}} = \theta\beta(1 + \alpha A) \frac{A'}{A} \frac{(1 + A) - \frac{\theta\beta(1 - \alpha)A}{1 - \beta}}{(1 + A') - \frac{\theta\beta(1 - \alpha)A'}{1 - \beta}}. \quad (46)$$

The following is established.

**Proposition 3:** *Assume that at the beginning of period  $T$  there is an unanticipated once-and-for-all productivity shock. The growth rate of the land price at period  $T$  is higher in the economy with binding credit constraint than without it if  $\theta\beta + \beta > 1$ , and the growth rate of the land price at period  $T + 1$  is smaller in the economy with binding credit constraint than unless the shock comes.*

*Proof:* The former part is straightforward from the analogous proof with the former part of Proposition 2. We prove the latter part by comparing  $g_{T+1}^Q$  in (45) and  $\theta\beta(1 + \alpha A)$ . We easily see  $g_{T+1}^Q$  in (45) is smaller than  $\theta\beta(1 + \alpha A)$  by calculation. Q.E.D.

We derive the interest rates, using (23) ( $R_t = \beta^{-1} g_t^Q$ ), as  $R_{T+2} = \theta(1 + \alpha A)$ ,

$$R_{T+1} = \theta(1 + \alpha A') \frac{(1 + A')}{(1 + A)} \frac{A}{A'}, \text{ and } R_T = \theta(1 + \alpha A) \frac{A'}{A} \frac{(1 + A) - \frac{\theta\beta(1 - \alpha)A}{1 - \beta}}{(1 + A') - \frac{\theta\beta(1 - \alpha)A'}{1 - \beta}}.$$

Figure 3, 4, and 5 illustrate the dynamic behavior of capital, the land price, and the real interest rate in the two economies in response to the productivity shock. The straight line illustrates the behavior of the economy with no binding credit constraint, and the dotted one with binding credit constraint. Any of three variables react only at period  $T + 1$  in the economy with no binding credit constraints, while all of them react at both period  $T$  and  $T + 1$  in the economy with binding credit constraint.



On impact,  $K_{T+1}$  increase in both economies. Additionally, the effect of credit mitigation is added in the economy with binding credit constraint. In response to the resulting rise in  $Q_T$ ,  $K_T$  increases. The growth-enhancing effect of the productivity shock is more persistent in the economy with binding credit constraint. Importantly, the impact of the shock begins one period ahead when credit constraints are binding.

One advantage of endogenous growth model is its tractability. We compare the magnitude of the cumulative effect of the shock between the two economies by calculating  $g_T^K g_{T+1}^K$  of each economy.

**Proposition 4:** *Assume that at the beginning of period  $T$  there is an unanticipated once-and-for-al productivity shock. The magnitude of the cumulative effect of the shock is greater in the economy with binding credit constraint than without it if*

$$\theta^2 \frac{(1 + \alpha A')}{(1 + A') - \frac{\theta\beta(1 - \alpha)A'}{1 - \beta}} > \frac{1 + \alpha A}{(1 + A) - \frac{\theta\beta(1 - \alpha)A}{1 - \beta}}. \quad (47)$$

Equation (47) is rewritten as  $\theta^2 \phi(A') > \phi(A)$ . Since  $\phi(\cdot)$  is increasing if  $\theta\beta + \beta > 1$ , this inequality is likely to be met. The growth-enhancing effect tends to be greater in magnitude in the economy with binding credit constraint than without it. Carlstrom and Fuerst (1997), following Bernanke and Gertler (1989), calibrate an infinitely-lived-agent model with binding credit constraint, and find that their agency-cost model replicates a sluggish and hump-shaped dynamics in investment in response to a productivity shock. Our finding shows that credit constraints do not necessarily become a source of sluggish dynamic in investment, but rather suggests that if the marketable asset is used as collateral, the economy with binding credit constraint may lead to the greater business fluctuations followed by a spike in investment, as in the standard RBC model.

## 6. Introduction of a Foreign Asset

We have thus far analyzed the closed economy. Now we introduce the foreign asset with a constant interest rate  $r$  into the model to satisfy  $\alpha A > r$ . This extension potentially allows investors to have another investment opportunity. If  $\theta(1 + \alpha A) \geq 1 + r$ , nothing changes because investors find it more beneficial to keep investing their funds in the domestic credit market than to invest abroad. If  $\theta(1 + \alpha A) < 1 + r$ , investors now find it more beneficial to invest abroad, and if the economy opens the capital account, the domestic interest rate will be driven up until it is equal to the world interest rate  $1 + r$ . It follows from (13) that the wealth of investors grow at rate  $\beta(1 + r)$ . The derivation of  $g^K$  does not need the market clearing in the credit market (12), and thus the growth rate of capital remains  $\theta\beta(1 + \alpha A)$ . Since it is smaller than the growth rate of the wealth of investors, the demand for credit is smaller than the supply of credit that investors can afford to make and the interest rate is kept to be tied with the world interest rate over time. The land price also grows at rate  $\theta\beta(1 + \alpha A)$ .<sup>4</sup> The equilibrium does not show a BGP. Finally, the difference from the closed economy is reflected in the capital-land-price ratio, which is given by

$$\frac{K_t}{Q_t} = \frac{(1 + \alpha A)\{(1 + r) - \theta\beta(1 + \alpha A)\}}{(1 + r)(1 - \alpha)A}. \quad (48)$$

The simple calculation shows that the capital-land-price ratio in the open economy is greater than that in the closed economy (given by (24)). By liberalization, the behavior of capital remains unchanged, and the rise in the interest rate leads to a decline in the land price level. The equilibrium is summarized in the following proposition

### Proposition 5:

*(Case A) Assume that  $\theta < 1$  and  $\theta(1 + \alpha A) \geq 1 + r$ . The economy is credit-constrained. The equilibrium exhibits a BGP with a growth rate  $\theta\beta(1 + \alpha A)$  and the interest rate  $\theta(1 + \alpha A)$ .*

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<sup>4</sup> The derivation is left to the Appendix.

(Case B) Assume that  $\theta < 1$  and  $\theta(1 + \alpha A) < 1 + r$ . The economy is credit-constrained. The equilibrium does not exhibit a BGP. The wealth of investors grows at the rate  $\beta(1 + r)$ , while capital grows at the rate  $\theta\beta(1 + \alpha A)$ . The interest rate is  $1 + r$ .<sup>5</sup>

By the introduction of a foreign asset into the model, the equilibrium with binding credit constraint is divided into two regimes. In one regime (Case A) the equilibrium shows a BGP with the endogenous determination of the interest rate, while in the other (Case B) the equilibrium does not show a BGP with the interest rate exogenously given. Assume that a permanent adverse shock to  $A$  happens when the economy is in the regime of Case A. As the productivity declines, the equilibrium leads to declines in the growth rate and the interest rate while keeping a BGP. If the productivity declines further beyond some critical level, the economy switches to the regime of Case B in which the growth rate of capital show a decline but the interest rate does not, and hence the equilibrium does not show a BGP.

In also Case B, we can investigate the impact of the once-and-for-all shock. As for the behavior of capital, the growth rates at  $T + 1$  and  $T + 2$  remain the same as Case A, but the growth rate at  $T$  is given by

$$g_T^K = \theta\beta(1 + \alpha A') \frac{(1 + A) - \frac{\theta\beta(1 - \alpha)A}{1 - \beta\theta(1 + \alpha A)/(1 + r)}}{(1 + A') - \frac{\theta\beta(1 - \alpha)A'}{1 - \beta\theta(1 + \alpha A')/(1 + r)}}, \quad (49)$$

which may or may not be greater than (44), depending on parameter values. Assume that  $A = 0.03$ ,  $A' = 0.04$ ,  $r = 0.01$ , and  $\beta = \theta = 0.98$ . The L.H.S. in (44) is greater than that in (48) if and only if  $\alpha < 0.58$ .

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<sup>5</sup> When the economy is not credit-constrained ( $\theta = 1$ ), the analysis is trivial. Since  $1 + \alpha A \geq 1 + r$  is always satisfied, nothing changes by the capital account liberalization.

## 7. Conclusion

In this paper we extend the K&M model to an endogenous growth model and investigate dynamic properties of a growing economy with binding credit constraint when land is used not only as an input of production but also as collateral. The developed model exhibits the propagation mechanism among output, capital, bank credit, and the land price in terms of the growth rate. The model's tractability allows us to derive interesting qualitative and quantitative findings. This analysis will be developed in several directions. First, it is interesting to extend the model to allow for investors to hold land, for example, for housing. The enriched model is expected to reproduce the observed behavior of the economy more accurately. Iacoviello (2005) calibrates the U.S. economy based on a version of the K&M model in which land is allocated between entrepreneurs and investors.

Second, it is worthwhile examining the possibility of debt renegotiation when the economy faces the adverse shock. When the adverse shock arrives, the liquidation value of land will depreciate below the promised debt repayment. The borrower will have an incentive of renegotiating debt repayment down until the depreciated liquidation value of land. This situation may be captured by the case of  $\Delta q_T = 0$  in Section 5, where the debt repayment is contingent not on the *anticipated* but on the *realized* land price. It will be valuable to pursue the asymmetry in the effects of the shock according to whether people face either the positive or the adverse shock. Sakuragawa and Sakuragawa (2007) investigate the VAR-based impulse responses and observe different impacts of the land price shock to other variables between before and after around 1990 in Japan.

Third, although the credit constraint is thought of one source of inertia of the economy in the literature of business fluctuations [e.g. Carlstrom and Fuerst (1997)], our finding show a contradictory finding that credit constraints do not necessarily become a source of inertia in investment, but rather may lead to the greater business fluctuations followed by a spike in

investment. This finding suggests that investment dynamics will be influenced by the marketability of assets used as collateral in the financial contract. This aspect of collateral may have an important insight on the role of credit constraints in business fluctuations

Fourth, it is interesting to examine the effects of government interventions. Sakuragawa and Sakuragawa (2006) study the impact of land taxation on the economy with binding collateral constraint and find several interesting results that would not arise in the economy with no constraint.

Fifth, it is interesting to extend the developed framework to a monetary model or multi-country model with international lending and borrowing. Our model with the endogenous determination of the interest rate will provide a useful framework to study the effect of the monetary policy or the integration of credit markets.

Finally, it is interesting to use the developed model to replicate the Japanese economy. To what degree our model reproduces the boom in the 1980s with appreciations in land prices and the slump in the 1990s with depreciations is an important topic.

#### Appendix: The Calculation of Section 6

It follows from (16) and  $R_{t+1} = 1 + r$  that

$$(1 - \alpha)AK_t = (1 + \alpha A)Q_t - \frac{1 + \alpha A}{1 + r}Q_{t+1} \quad (\text{A-1})$$

It follows from (A-1) and (22) that

$$\frac{(1 + \alpha A)Q_t - \frac{1 + \alpha A}{1 + r}Q_{t+1}}{(1 + \alpha A)Q_{t-1} - \frac{1 + \alpha A}{1 + r}Q_t} = \theta\beta(1 + \alpha A) \quad (\text{A-2})$$

Using  $g_{t+1}^Q \equiv \frac{Q_{t+1}}{Q_t}$ , rearrangement leads to

$$g_{t+1}^Q = (1 + r) + \theta\beta(1 + \alpha A) - \theta\beta(1 + \alpha A)(1 + r)(g_t^Q)^{-1}. \quad (\text{A-3})$$

When  $g^Q \equiv g_t^Q = g_{t+1}^Q$ ,  $g^Q = \theta\beta(1 + \alpha A)$ , or  $= 1 + r$ . Since  $g^Q = 1 + r$  violates the transversality condition,

$$g^Q = \theta\beta(1 + \alpha A) \quad (\text{A-4})$$

is the unique solution. Finally, it follows from (A-1) and (A-4) that

$$\frac{K_t}{Q_t} = \frac{(1 + \alpha A)\{(1 + r) - \theta\beta(1 + \alpha A)\}}{(1 + r)(1 - \alpha)A}. \quad (\text{A-5})$$

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**Figure 1: The sequence of events**

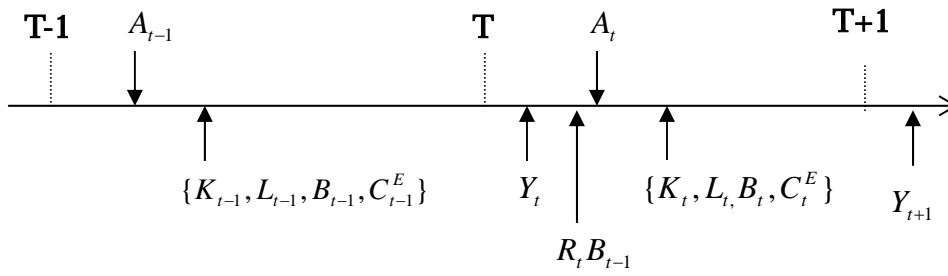
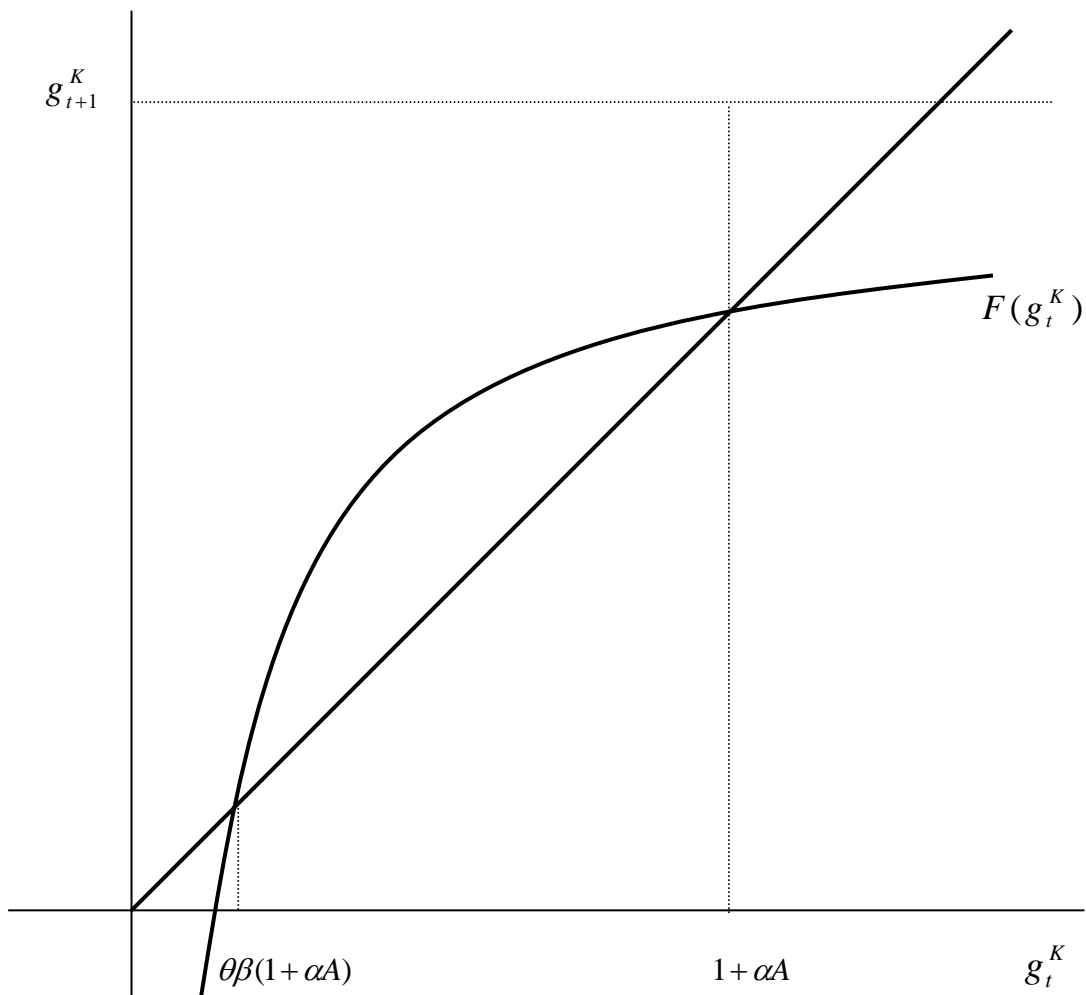
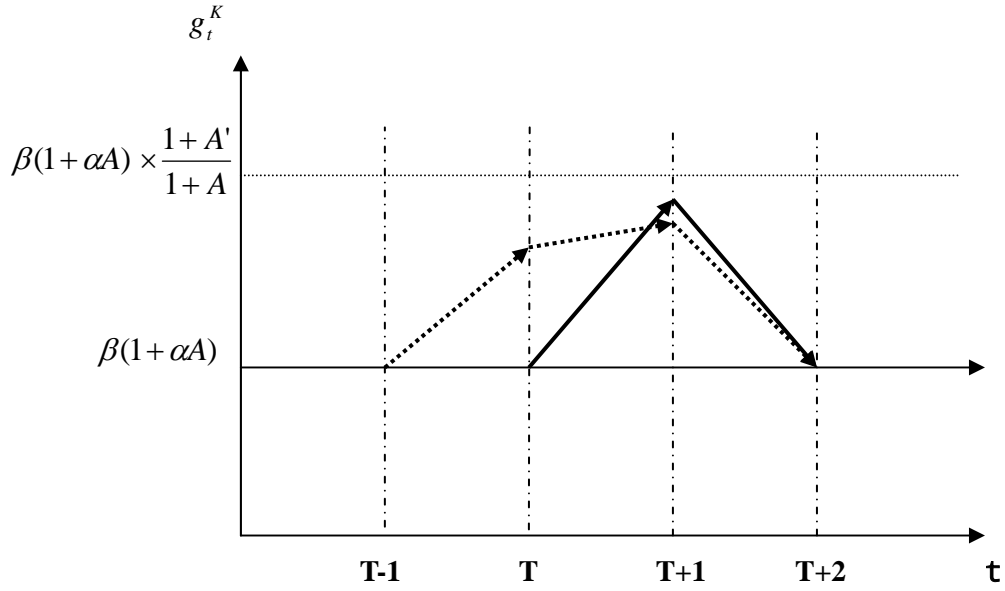




Figure 2: The Existence of the Growth Path

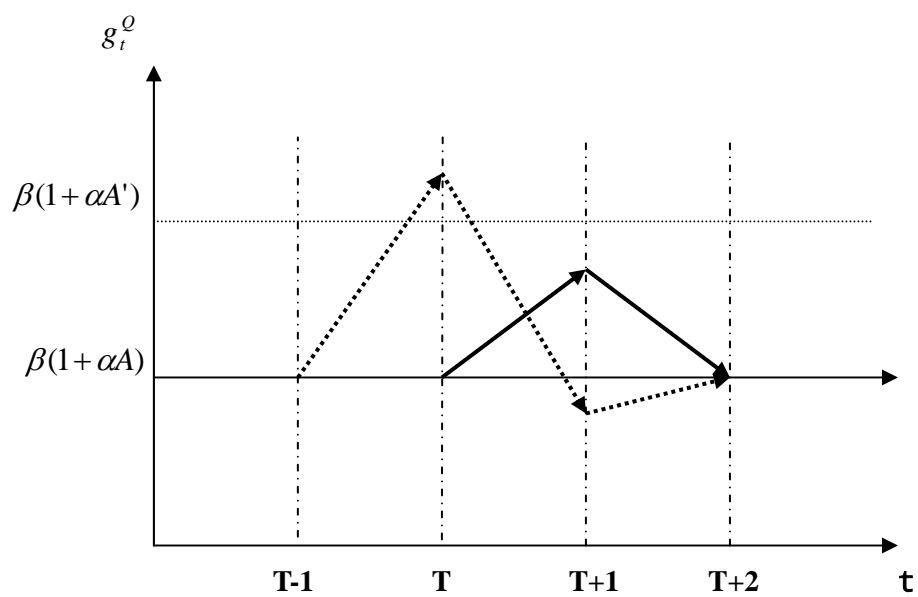


**Figure 3 : Dynamic behavior of Capital in response to productive change**



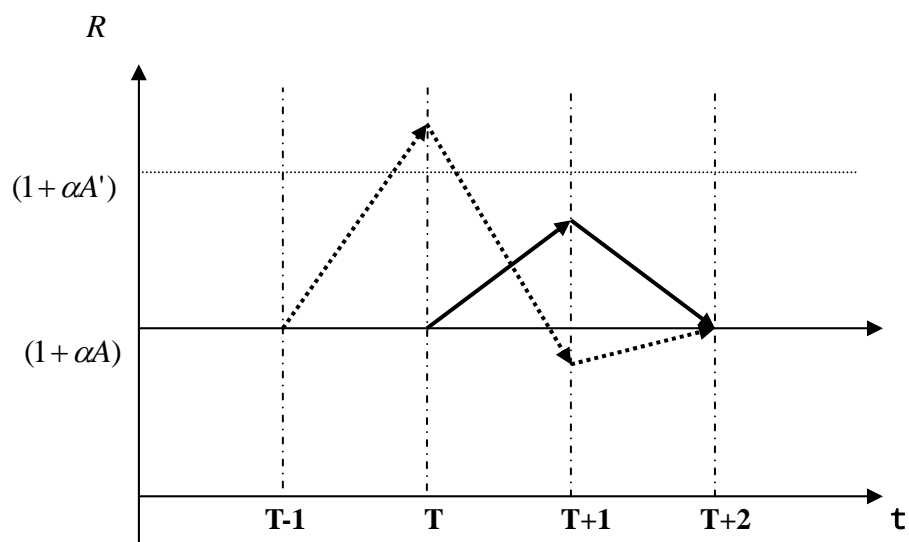
Note) The straight line illustrates the behavior of the economy with no binding credit constraint, and the dotted one the behavior of the economy with binding credit constraint.

Figure 4 : Dynamic behavior of the land price in response to the productive change



Note) The straight line illustrates the behavior of the economy with no binding credit constraint, and the dotted one the behavior of the economy with binding credit constraint.

**Figure 5 : Dynamic behavior of the interest rate in response to the productive change**



Note) The straight line illustrates the behavior of the economy with no binding credit constraint, and the dotted one the behavior of the economy with binding credit constraint.