

Gender Segregation of Skill Acquisition: Theory and Policy Implications*

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Abstract

This paper presents a model that can account for the gender segregation of skill acquisition when the marriage market is competitive. We in particular show that when the burden of domestic activities, arising most notably from childbearing and child rearing, is asymmetrically placed on married women, there arises an incentive for them to deliberately degrade the market value of acquired skills. We then show that this incentive can be excessively strong and gives rise to the emergence of an inefficient asymmetric equilibrium where a bulk of women concentrate on acquiring skills that do not lead to higher wages in the labor market. The analysis reveals why policy interventions such as affirmative action programs or equal employment opportunity laws that directly subsidize the acquisition of skills would not be effective in closing the gender earnings gap in the long run, and instead suggests simple alternative measures to correct this distorted system of incentives.

JEL Classification Codes: J12, J24.

Key Words: Gender segregation; Human capital investment; Marriage; Intrahousehold Bargaining; Affirmative action.

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1 Introduction

Personal attributes often dictate the way people acquire skills. Among them, gender in particular seems to be a crucial determinant of the pattern of skill acquisition. There is a worldwide trend that women do not invest in skills that directly lead to higher wages in the labor market as much as men do in terms of both quantity and quality. First, in many countries, women tend to lag behind men in terms of educational attainment (Echevarria and Merlo 1999, Figure 1). Second, even in countries where the gender difference in educational attainment no longer exists, the pattern of skill acquisition has in general been highly segregated by gender. In the US, for instance, women have been under-represented in high-income majors, such as engineering and business, at least until recently: they made up only 9 percent of all business majors and 0.8 percent of all engineering majors in 1971 while they made up 74 percent of all education and foreign language literature majors (U.S. Census Bureau, 2004-2005, No.285). The asymmetric pattern of skill acquisition is apparently not an isolated phenomenon in the US. In Japan, besides the gender difference in college major choices, a substantial portion of women attend two-year junior colleges which place clear emphasis on the acquisition of domestic skills such as home economics or domestic science.¹ In many other countries, gender also seems to play a decisive role in the type of skills to acquire (see Table 1).

From the purely theoretical point of view, this asymmetric pattern of skill acquisition can be seen as a way to maximize the benefit of role specialization within households. As Becker (1991) suggests, the two parties in a household generally do not need to acquire the same set of skills: if men invest in skills designed for market activities, it is often more beneficial for women to acquire skills designed for domestic activities.² In order to reap this benefit of role specialization, the pattern of skill acquisition can be highly segregated by gender. Although Becker analyzes a situation where the investments take place after marriage, subsequent studies such as Echevarria and Merlo (1999), Hadfield (1999), Engineer and Welling (1999) and Ishida (2003) all confirm in various settings that Becker's original insight still holds even if the investments take place before marriage. In this line of research, the main issue is intrahousehold coordination where the

¹The gender difference in the college enrollment rate had disappeared in Japan by the late 1980s. Of those women, however, about 60% attended junior colleges at the time while it was only 5% for men (Ministry of Education, Culture, Sports, Science and Technology of Japan, 2004).

²Throughout the paper, the term 'domestic activities' is broadly referred to as various non-market activities such as house-keeping, childbearing and child rearing.

anticipation of future role specialization results in the asymmetric pattern of skill acquisition.

While those previous studies are certainly suggestive, there is an important caveat for their results: those models assume either that agents are homogenous before investment decisions or that the matching is random.³ These assumptions may trivialize a critical aspect of marriage formation since the matching pattern in the marriage market is normally positively assortative. Recently, several models explicitly incorporate competition for matching partners to analyze situations where investment decisions explicitly affect the type of matching partner (Cole et al. 2001a,b; Peters and Siow 2002). Those models clarify that agents in general have stronger incentives to invest when they have concerns about their future matching partners. As this competition effect provides an additional incentive to invest, the investment levels become efficient in some equilibria despite the holdup property of the underlying environment. With the competitive marriage market where agents compete for marital partners, standard models are no longer capable of replicating the strongly asymmetric pattern of skill acquisition between men and women (Rios-Rull and Sanchez-Marcos 2002).⁴

Given this recent theoretical development, the present paper first presents a mechanism that can account for the asymmetric pattern of skill acquisition in the presence of the competitive marriage market and then derives welfare and policy implications from it. To this end, we construct a model where agents who are heterogeneous in ability must invest in skills before marriage. Since the two matched agents in a household cannot in general write a binding contract *ex ante*, the nature of intrahousehold resource allocation becomes critical as it links the two markets of our interest – the labor market where surplus is generated and the marriage market where the surplus is redistributed within each household. To be consistent with recent empirical evidence, we assume that the total surplus is divided through the process of intrahousehold (Nash) bargaining.⁵ The way the total surplus is redistributed within each household evidently affects the pattern

³The random matching assumption implies that marriages are formed based on exogenous (noneconomic) factors: Engineer and Welling (1999) refer to this as the ‘true-love’ criteria. In this paper, we take the opposite stance that marriages are formed based purely on endogenous (mostly economic) factors. These views clearly represent the two extreme points of the spectrum, and reality must lie somewhere in-between. It should be noted that our main results hold even when exogenous factors play some role in marriage formation, as long as endogenous factors are sufficiently important.

⁴Among several possibilities, they conclude that the most successful model is the one where parents strive to maximize the number of grandchildren. A problem with this approach, which relies on parental preferences, is that it does not explain why the gender gap in educational attainment has steadily been shrinking in some developed countries, the point we will also explore in this paper.

⁵In many economic analyses, households are often considered as the minimum decision-making unit: it is typically assumed

of skill acquisition.

Within this framework we first show that there exists an inefficient asymmetric equilibrium where a substantial portion of women intentionally acquire skills that are less marketable (in the labor market). The logic behind this result is as follows. With marriage arises the benefit of role specialization. As full-time jobs normally require full-time effort, it is often optimal for at least one member of the household to focus exclusively on market activities. In many case, it is women who expend more resources on domestic activities since they often possess comparative advantage in them (Lazear and Rosen 1990; Echevarria and Merlo 1999). Provided that the productivity in domestic activities does not depend strongly on the level of accumulated skills, this implies that at least some fraction of women's skills must be wasted when they are married. Women with more earnings potential are then not desirable as marital partners from the viewpoint of men because those women have stronger bargaining power without increasing the total surplus of the household sufficiently. In this case, therefore, women face an interesting tradeoff: while they can raise their wages by acquiring marketable skills, this may actually work to their disadvantage in the marriage market because their bargaining power may become excessively strong. This strategic aspect of skill acquisition may distort women's incentives to acquire skills and lead then to intentionally acquire less marketable skills. We show that this distorted incentive is excessively strong from the social point of view and leads them to overinvest in skills that do not lead to higher wages in the labor market.

Whether this inefficient asymmetric equilibrium arises depends on the magnitude of the cost of domestic activities. Women with more marketable skills are less preferred by men when the cost of domestic activities is large and women need to devote a significant fraction of resources for domestic activities once married. As the burden of domestic activities becomes less significant and the opportunity cost of marriage decreases, the situation is turned around at some point and there arises a symmetric equilibrium where the gender difference in the pattern of skill acquisition disappears.⁶ The present analysis thus reveals that the source

that each household acts as a single decision-making unit and each member of a household earns the same level of utility. Recent evidence seems to indicate that this pooled income approach is empirically not consistent (Thomas 1990; Browning et al. 1994; Chiappori et al. 2002).

⁶We argue that this is roughly consistent with the recent trend in the US. As stated, in the US, women made up only 9 percent of all business majors and 0.8 percent of all engineering majors in 1971. In 2002, the fractions of female business and engineering majors have risen to 50 and 19 percent, respectively. The same trends can be observed for the gender difference in occupational choices. Black and Juhn (2000, Table 1) show that the fraction of women in high-wage occupations, defined as

of the inefficiency lies in the asymmetric cost structure of domestic activities, which leads to the earnings differential between single and married women. This logic indicates that simply subsidizing women for the acquisition of skills is not effective because such a policy intervention benefits both single and married women proportionally and hence has no impact on the pattern of investment. What is expected to be more effective is an income transfer program that compensates married women for the opportunity cost of marriage (or alternatively the lost market income) as it can effectively reduce the earnings differential. In other words, a policy intervention which shifts the cost of domestic activities asymmetrically placed on married women to all members of the society, regardless of gender and marital status, can have a drastic effect on the pattern of skill acquisition and consequently the gender difference in earnings.⁷ The main message of this paper is that when it comes down to closing the gender gap in earnings, one needs to pay close attention to the gap between single and married women, rather than that between men and women. In light of this view, we argue that policy interventions such as paid maternity leaves, child care benefits, or subsidies to nursery schools are much more effective in closing the gender gap in the long run than affirmative action programs or equal employment opportunity laws that directly subsidize the acquisition of skills for all women.

Besides this policy implication, the present paper also raises several theoretical issues. In our model, the interactions of three contributing factors – assortative matching, intrahousehold bargaining, and the asymmetric cost structure – are crucial in giving rise to the distorted system of incentives. To see this, it is important to note that, as already stated, competition in the matching market (assortative matching) typically has a positive incentive effect. Note also that the presence of intrahousehold bargaining by itself provides an additional incentive, compared to the case where the total income of a household is pooled and equally shared among its members, because agents can raise their threat point by acquiring skills.⁸ When these two factors are combined with the asymmetric cost structure, however, they are turned into negative incentives which lead women to invest inefficiently in order to reduce their threat point and make themselves more attractive in the marriage market. In this respect, our result strongly stands in contrast with the

the top twenty percent of jobs in terms of male wages, have increased from 8 percent in 1967 to 24 percent in 1997. Changes in the pattern of skill acquisition have narrowed the gender earnings gap (Blau and Kahn 1992; O'Neill and Polachek 1993).

⁷Empirical evidence shows that differences in college major choices, and the consequent differences in occupation choices, were major sources of the gender earnings gap (Brown and Corcoran 1997; Altonji and Blank 1999).

⁸Models such as Echevarria and Merlo (1999), Konrad and Lommerud (2000), Lundberg and Pollak (2003) and Ishida (2003) all share this aspect.

previous literature on *ex ante* investment with intrahousehold bargaining where agents are homogeneous and hence assortative matching is not an issue. This inefficiency result is also distinct from previous studies which mainly focus on the holdup problem. For instance, Peters and Siow (2002) show that competition in the marriage market is instrumental in resolving the holdup problem and leads to the efficient allocation under certain conditions. In contrast, we show that competition in the marriage market can be the source of a different type of inefficiency.⁹

The rest of the paper is organized as follows. In section 2, we first offer some discussion on the cost structure of domestic activities which, along with the process of intrahousehold bargaining, plays a critical role in our analysis. In section 3, we briefly outline the model. In section 4, we characterize equilibria and show that there arise two distinct types of equilibrium, asymmetric and symmetric, depending on the relative cost of domestic activities. In this process, we also illustrate the underlying mechanism by which women invest in intrinsically useless skills. In section 5, given the results obtained in the previous section, we discuss key properties of the model. We in particular argue that the investment pattern of women tends to be inefficient in the asymmetric equilibrium and offer a potential remedy for it. Finally, in section 6, we make some concluding remarks.

2 On the cost structure of domestic activities

One of the most critical aspects of the model is the earnings differential between single and married women (hereafter, we sometimes refer to this simply as the earnings differential), which reflects the opportunity cost of marriage for women. The earnings differential arises from the fact that the burden of domestic activities is asymmetrically placed on married women. The basic presumption behind this is that women possess comparative advantage in domestic activities and thus tend to spend more time and effort for them once they are married. Since this assumption plays such a critical role in the subsequent analysis, we first elaborate on why this differential arises in some detail in order to clarify the scope of the paper.

There are several channels through which this comparative advantage arises. First and foremost, there is a biological restriction that only women can bear children. This inherent gender difference lowers their

⁹Nosaka (forthcoming) shows another source of inefficiency when the utility is submodular. In this case, the negatively assortative matching is more efficient, but the competition effect prevents this matching formation.

productivity in market activities in many ways. For instance, Corcoran and Duncan (1979), Mincer and Ofek (1982), Cox (1984) and Lazear and Rosen (1990) all emphasize the connection between career interruptions and earnings growth for women. Moreover, Echevarria and Merlo (1999) construct a dynamic household bargaining model and show that when the cost associated with bearing children is positive, then in equilibrium women also bear the entire cost associated with rearing children. They then estimate the cost to a woman of having a child is roughly 5% of her working lifetime.

One can also point out various ways in which women's comparative advantage in domestic activities may be endogenously created. Since women have traditionally had higher turnover than men, it is intuitively clear that firms provide more internal training to men as often emphasized. On the other side of the coin, since domestic activities are predominantly conducted by women, it is also likely that firms providing the technology of domestic activities focus more on their needs. In either case, it is natural that women devote more resources to domestic activities. The assumption on the earnings differential can then be viewed as a reduced form of this structure.

In this paper, we take as given that women possess comparative advantage in domestic activities partly because there are so many works on gender differences that arise endogenously. Given this, our goal in this paper is to show how this (possibly slight) quantitative gender difference is amplified into large qualitative differences in behavioral outcomes. The important point of our argument is that women tend to specialize more heavily in domestic activities once they are married in order to allow their spouses to concentrate on market activities. This expected pattern of role specialization certainly lowers the returns to skills when married and, more importantly, results in the earnings differential. Since married women cannot fully utilize their accumulated skills, their lost market income must somehow be compensated within households. From the viewpoint of potential partners, those women with more marketable skills are perceived to possess excessively strong bargaining power and thus to be less desirable as marital partners. In what follows, we formalize this intuition and provide a mechanism which leads to the asymmetric pattern of skill acquisition under the competitive marriage market.

3 The model

3.1 Environment

Consider a two-period model where there is a continuum of agents who belong to either one of the two gender subsets of equal size, each denoted by $j \in \{f, m\}$ ($j = f$ for female and $j = m$ for male). Agents acquire skills in the first period and search for marital partners in the second. If two agents decide to marry, they form a household where they bargain over the total surplus.

Each agent is completely characterized by the ability type $x \in [0, \bar{x}]$ and gender $j \in \{f, m\}$. The ability type is drawn randomly from some distribution F , which is independent of gender. We make the following assumptions concerning the distribution of the ability type.

Assumption 1 *F is continuously differentiable and strictly increasing in $x \in [0, \bar{x}]$. In addition, the upper-bound of the support \bar{x} is sufficiently large.*

3.2 Skill acquisition

Agents can accumulate skills along two dimensions, the level $e_j(x) \in R_+$, and the market value $q_j(x) \in [0, 1]$. The market value simply reflects differences in the nature of skills, and is totally independent of how difficult or costly it is to acquire those skills.

The market productivity is the product of the two elements, $q_j(x)e_j(x)$. Let $h_j(x) = (q_j(x), e_j(x)) \in [0, 1] \times R_+$ denote the investment choice. For notational simplicity, we sometimes write this as $h_j = (q_j, e_j)$ or simply $h = (q, e)$ wherever it is not confusing. Given some ability type x , the cost of acquiring skills is a function of the investment level e but not of the market value q :

$$C(e, x) = \frac{\beta}{x(1 + \gamma)} e^{1+\gamma}, \quad \gamma > 0, \quad (1)$$

where $\beta > 0$ is the parameter that measures the relative cost of skills. This specification implies that there is no cost in upgrading the market value of skills, and the social efficiency thus requires $q_j(x) = 1$ for all x .

3.3 Marriage and production

In the second period, each agent decides whether to enter the marriage market, which is assumed to be competitive. More precisely, we mean by the competitive marriage market that the resulting matching pattern must be stable in the sense that no matched pair has an incentive to unilaterally dissolve the marriage in search of another partner. This implies that the resulting matching pattern is positively assortative (in a sense to be made more precise below).

If an agent decides not to enter the marriage market, the agent remains single and concentrates on market activities. The total utility when an agent remains single is equal to the market productivity qe regardless of gender.

If an agent decides to enter the marriage market and finds a partner, the two matched agents form a household in the second period. The gains from marriage arise from the investment choices made in the first period. Consider a household where the investment choice for the female agent is h_f and that for the male agent is h_m . We then specify that the joint outcome for this household is given by

$$y(h_f, h_m) = (1 + \alpha) \left(q_m e_m + (1 - \theta) q_f e_f \right) + 2D(e_f, e_m) - 2d, \quad \theta \in (0, 1). \quad (2)$$

The benefit of marriage arises from two sources, the household public good captured by the first term and other (mostly psychological) factors captured by the second. There is also a fixed cost of marriage (d for each agent) which leads some agents to remain single.

The first term represents the material gain from marriage, which is entirely determined by the market income of the household. The fact that $\theta > 0$ implies that the returns to skills are lower by design for married female agents because they must devote some resources to domestic activities, as assumed in Echevarria and Merlo (1999). Notice that θ captures the opportunity cost of marriage for female agents, which leads to the earnings differential with respect to the marital status. The female agent in a household spends a fraction θ of her endowed time to produce the household public good, such as well brought-up children and a clean and tidy household. The total market income is thus multiplied by $1 + \alpha$, $\alpha > 0$, due to the production of the household public good.¹⁰ The larger the multiplier α is, the more important the public good is for the household. Throughout the analysis we consider the case where α is sufficiently large relative to its cost.

¹⁰When there is a household public good, the agents in a household can enjoy more consumption with the same level of income due to the non-excludability of the public good (Lam 1988). For example, suppose that the utility function is

More specifically, we assume that the following condition is satisfied:

Assumption 2

$$\left(\frac{\alpha}{2} - \gamma\right)\left(\frac{\alpha}{2} + 1\right)^{\frac{1}{\gamma}} \geq 1 \quad (3)$$

This assumption is purely technical and qualitatively inconsequential: it simply assures that the value of marriage is sufficiently large so that agents actually have an incentive to marry.

Aside from this material benefit, there is also a (psychological) benefit from marriage, which is positively related to the level of skills. This component is captured by the second term $D(e_f, e_m)$ where

$$D(e_f, e_m) = \delta(e_f + e_m). \quad (4)$$

We assume that the psychological benefit does not depend on the market value of skills q_j . This captures the fact that in order to enrich marriage lives, many different types of knowledge and skills are valuable, including those that are less productive in the labor market. The market value of skills q_j is no longer appropriate to measure the impact of skills in this particular sense.

For expositional clarity, we sometimes refer to δ as the intrinsic value of skills, partly to contrast with the endogenously chosen market value $q_j(x)$. We are generally interested in the case where the intrinsic value of skills is relatively small.¹¹

3.4 Intrahousehold bargaining

Since the private good is transferable, the agents in a household negotiate over how to divide the total surplus. The outcome of the negotiation is characterized by the Nash bargaining solution where the threat $U_i = \min[\alpha I, Y] + X_i$ in which Y , I , and X_i are the household public good, the household market income, and the private good for agent i . The budget constraint for the household is given by $Y + X_f + X_m = I$. Since the private goods are transferrable, the household maximizes the total utility $U_f + U_m = 2 \min[\alpha I, Y] + I - Y$. It is immediate to see that the total utility is maximized when $Y = \alpha I$, and the indirect utility is thus given by $U_f + U_m = (1 + \alpha)I$. We can derive a similar utility from more general cases such as $U_i = a(Y)X_i$, but we need a linear approximation in these cases.

¹¹It is important to note that we certainly do not intend to imply that it is socially meaningless to acquire skills with less market value as those skills can benefit the society in many tangible and intangible ways. We simply attempt to show that there are situations in which the incentive to acquire those skills can be excessively strong because the incentive arises independently of their intrinsic values.

point for each agent is the total utility when the agent remains single, which is simply given by the skill level in the market (note that there is no household public good in this case).¹² The formula for the Nash solution produces the bargaining outcome $V_j(h_f, h_m)$ for each agent as a function of the investment choices:

$$\begin{aligned} V_j(h_f, h_m) &= \frac{1}{2} \left(y(h_f, h_m) - q_f e_f - q_m e_m \right) + q_j e_j. \\ &= \frac{\alpha}{2} \left(q_m e_m + q_f e_f \right) + \delta(e_f + e_m) - d + q_j e_j - \frac{\theta(1 + \alpha)}{2} q_f e_f. \end{aligned} \quad (5)$$

4 Equilibrium

We now characterize the equilibrium of the model described above. Throughout the analysis, we confine our attention to separating equilibria where the level of investment is strictly increasing in the ability.¹³

4.1 The marriage market

To solve the model backwards, we first characterize the preferences for marital partners for each gender subset. For this purpose, we modify (5) as follows:

$$V_m(h_f, h_m) = \left(\left(\frac{\alpha}{2} + 1 \right) q_m + \delta \right) e_m + \delta a_f(h_f) - d, \quad (6)$$

$$V_f(h_f, h_m) = \left(\frac{\alpha}{2} + \delta \right) a_m(h_m) + \left((\Delta + 1) q_f + \delta \right) e_f - d, \quad (7)$$

where

$$\Delta \equiv \frac{\alpha - \theta(1 + \alpha)}{2}.$$

In these expressions, a_j is what we call the attractiveness, which is defined as

$$a_f(h_f) = \frac{1}{\delta} (\Delta q_f + \delta) e_f, \quad (8)$$

$$a_m(h_m) = \left(\frac{\alpha}{2} + \delta \right)^{-1} \left(\frac{\alpha}{2} q_m + \delta \right) e_m. \quad (9)$$

¹²In other words, we view the ultimate threat point of intrahousehold bargaining as a divorce. An alternative approach is to view it as a noncooperative marriage. See Lundberg and Pollak (1993, 1994) for this approach.

¹³In general, we cannot rule out the possibility of partial pooling where a subset of agents choose the same investment levels. Although it is possible to exclude this possibility with additional restrictions on equilibrium, we do not pursue this issue because it is purely technical without yielding much economic insight.

The attractiveness represents the gross benefit that an agent can give to the marital partner. The preferences for partners are thus entirely summarized in a scalar variable a_j . It is immediate to see from this specification that more attractive agents are more desirable as marital partners.

There are two critical implications drawn straightforwardly from this. First, female agents always prefer male agents with higher q_m , i.e., more marketable skills. While male agents with more marketable skills have stronger bargaining power, this negative effect is totally dominated by the higher income earned by them because they can devote all of their resources to market activities. Second, on the other hand, an increase in the market value of skills may or may not increase the attractiveness of female agents. The deciding factor is the sign of Δ , which subsequently determines the nature of equilibrium. We thus need to study two distinct cases, depending on the sign of Δ :

Condition D $\Delta < 0$ or, equivalently, $\frac{\alpha}{1+\alpha} < \theta$.

When Condition D holds, female agents are not sufficiently productive because they need to spend a significant amount of time for domestic activities once married. Female agents cannot fully utilize their acquired skills, and their bargaining power thus becomes excessively strong from the viewpoint of male agents. Because of this, there may arise an incentive for them to intentionally degrade the market value of skills in order to secure the benefit of marriage. As we will see shortly, this incentive gives rise to the emergence of what we call the asymmetric equilibrium where male and female agents behave differently in a qualitative sense.

4.2 Asymmetric equilibrium

The argument made thus far implies that the equilibrium investment choice depends critically on whether Condition D holds. We first consider the case where Condition D holds. Under this condition, female agents with more marketable skills are less preferred by male agents in the marriage market. This force leads to the emergence of the asymmetric equilibrium where a significant fraction of female agents choose the lowest market value even when it is costless to upgrade it.

In order to examine the incentives to invest, we first define a return function ϕ where $\phi(a_f)$ denotes the attractiveness of the husband with which each female agent with attractiveness a_f expects to match.¹⁴

¹⁴Although the characteristics of agents are two dimensional (q_j and e_j), we can still take this approach, because the

In order for the match formation to be stable, we need the resulting matching pattern to be positively assortative. In terms of the return function ϕ , the stability requires that ϕ be a strictly increasing function of a_f . Given this, we can now define

$$\begin{aligned} U_m(h_m, x) &\equiv V_m(h_m, \phi^{-1}(a_m)) - C(e_m, x), \\ &= \left(\left(\frac{\alpha}{2} + 1 \right) q_m + \delta \right) e_m + \delta \phi^{-1}(a_m(h_m)) - d - C(e_m, x), \end{aligned} \quad (10)$$

$$\begin{aligned} U_f(h_f, x) &\equiv V_f(\phi(a_f), h_f) - C(e_f, x) \\ &= \left(\frac{\alpha}{2} + \delta \right) \phi(a_f(h_f)) + \left((\Delta + 1) q_f + \delta \right) e_f - d - C(e_f, x), \end{aligned} \quad (11)$$

taking the return functions $a_m = \phi(a_f)$ and $a_f = \phi^{-1}(a_m)$ as given. With this specification, each agent in gender subset $j = f, m$ chooses h_j to maximize $U_j(h_j, x)$. Solving these problems we can make the following statement.

Proposition 1 (Asymmetric equilibrium) *Suppose that (i) condition D holds and (ii) δ is sufficiently close to zero. Then, there exists an asymmetric equilibrium where agents choose to marry if and only if $x \geq x^*$ for some $x^* \in (0, \bar{x})$. In the limiting case where $\delta \rightarrow 0$, the optimal investment choices in the asymmetric equilibrium, denoted by $h_j^{\text{asym}}(x) = (q_j^{\text{asym}}(x), e^{\text{asym}}(x))$, are given by*

$$\begin{aligned} q_f^{\text{asym}}(x) &= 0, \quad q_m^{\text{asym}}(x) = 1, \\ e_f^{\text{asym}}(x)^{1+\gamma} &= \frac{\alpha}{2} \left(\frac{\alpha}{2} + 1 \right)^{\frac{1}{\gamma}} \left(\frac{x}{\beta} \right)^{\frac{1+\gamma}{\gamma}} - \gamma \left(\frac{\alpha}{2} + 1 \right)^{\frac{1}{\gamma}} \left(\frac{x^*}{\beta} \right)^{\frac{1+\gamma}{\gamma}}, \quad e_m^{\text{asym}}(x)^\gamma = \left(\frac{\alpha}{2} + 1 \right) \frac{x}{\beta}, \end{aligned}$$

for $x \geq x^*$.

Proof. See Appendix.

Here, we assume $\bar{x} > x^*$ by the latter part of Assumption 1 so that the most able agent actually has an incentive to marry. In equilibrium, since the ability distribution of the male population is identical to that of the female counterpart, any matched pair consists of two agents of the same type (positive assortative matching).

Under condition D, there arises an interesting tradeoff for female agents in that an increase in the market value leads to a decrease in the attractiveness, as can be seen from (8). There are thus two ways to achieve a

 preferences of agents are summarized in the scalar variable a_j .

given level of the attractiveness, either to increase the investment level or to decrease the market value. This tradeoff is summarized in Figure 1, which illustrates the indifference curves for male and female agents.¹⁵ While the indifference curve for male agents is standard and convex to the origin, that for female agents is more complicated as the market value varies with the targeted attractiveness. Intuitively, female agents choose a positive market value when the targeted attractiveness is relatively low due to the increasing marginal cost. As the targeted attractiveness increases, the marginal cost of investment becomes too high, and they start to lower the market value instead. In particular, when the targeted attractiveness gets past some threshold level, denoted as $e_f^u(x)$ in the figure, it becomes optimal for them to choose $q_f(x) = 0$.¹⁶ Intuitively, female agents sacrifice their market income, when this loss is compensated by the improvements in the quality of matching partners. Notice that this incentive becomes stronger and leads them to choose $q_f(x) = 0$ when δ is sufficiently small, as shown in the proposition. Provided that the cost of skill acquisition is totally independent of the market value, this incentive entails a pure welfare loss. This fact leaves a room for some government interventions, which will be discussed later in section 5.3.

[Figure 1 about here]

4.3 Symmetric equilibrium

We now consider the case where Condition D fails to hold, and the attractiveness is strictly increasing in the market income for both gender subsets. There is thus no incentive to degrade the market value of skills even for female agents. With $q_f(x) = q_m(x) = 1$ for all x , the problem is greatly simplified. Since the optimization problem is defined over a single variable e_j , we only need to consider the correspondence between e_f and e_m , instead of that between a_f and a_m , without loss of generality. Let $e_m = \psi(e_f)$ denote a return function defined on e_j . We then focus on the symmetric case in which $e = \psi(e)$.

Proposition 2 (Symmetric equilibrium) *Suppose that condition D does not hold. Then, there exists a symmetric equilibrium where agents choose to marry if and only if $x \geq x^{**}$ for some $x^{**} \in (0, \bar{x})$. The*

¹⁵For the details of the indifference curves, see Appendix, especially lemma A.1.

¹⁶Although not explicitly written in the proposition, there are some female agents, those with $x < x^*$, who remain single and choose $q_f(x) = 1$.

optimal investment choices in the symmetric equilibrium, denoted by $e_f^{\text{sym}}(x)$ and $q_j^{\text{sym}}(x)$, are given by

$$q_f^{\text{sym}}(x) = q_m^{\text{sym}}(x) = 1,$$

$$e_f^{\text{sym}}(x)^\gamma = e_m^{\text{sym}}(x)^\gamma = \left(1 + 2\delta + \frac{\alpha}{2} + \frac{\alpha - \theta(1 + \alpha)}{2}\right) \frac{x}{\beta},$$

for $x \geq x^{**}$.

Proof. See Appendix.

We once again assume $\bar{x} > x^{**}$ so that the most able agent actually has an incentive to marry. The analysis of this symmetric case is fairly standard (for example, Peters and Siow, 2002). Although the burden of domestic activities is still asymmetrically placed on female agents, the process of intrahousehold bargaining results in the equal share of the net total surplus between male and female agents, if their threat points are identical. Due to this symmetric property, the equal investment levels ($e_f^{\text{sym}}(x) = e_m^{\text{sym}}(x)$) are supported in equilibrium, even though the social returns to skills are not identical. Moreover, the competition effect is stronger and hence induces higher investment levels than in the asymmetric equilibrium since female agents are more diverse in terms of the attractiveness.

5 Discussion

5.1 Comparative statics

The previous section establishes that the resulting equilibrium pattern can change drastically as the parameters change. A technological progress is especially a source to affect these parameters over time. First, consider the effect of an improvement in the productivity of the household public good, which can be seen as a reduction in θ . Apparently, a change in θ has a significant impact on whether Condition D holds and thus on the resulting equilibrium pattern. In general, an improvement in the productivity of the household public good allows female agents to shift their endowed resources more heavily toward market activities and thereby reduces the earnings differential. At some point, an investment in the market value of skills becomes sufficiently profitable for female agents and this leads to the emergence of the symmetric equilibrium. In the US, there has been a steady increase in the proportion of female college students choosing high-income

majors such as engineering and business (see footnote 6). The model's prediction is largely consistent with this recent trend.

Under the present setup, on the other hand, a reduction in the cost of acquiring skills, i.e., a decrease in β , has different implications. A change in β could have some qualitative effects on the equilibrium level of skills even when the asymmetric equilibrium prevails. First, it raises the marginal value of skills, and the incentive to invest in them becomes stronger for male agents. Second, more investment from male agents implies more intense competition among female agents which also leads them to invest more. It is clear, however, that the nature of equilibrium is totally independent of β in a qualitative sense as Condition D is totally independent of it. This implies that, as long as Condition D holds, it remains to be the relevant investment choice for female agents to degrade the market value of skills to the lowest level regardless of β .

In this framework, a reduction in the cost of acquiring skills (or, equivalently, an improvement in the productivity of skills) has no impact at all on the nature of equilibrium. Apparently, this rather strong result is a figment of our model specification to some extent and we certainly do not claim this to be a general result. We still argue, however, that this result reveals something fundamental behind this whole process. The important point is that a reduction in the cost of acquiring skills has no effect on the investment pattern because its effect (as designed in this model) is neutral as to the marital status: a decrease in β raises the market income proportionally for both single and married agents. This is not the case for an improvement in the productivity of the household public good. In our model, a decrease in θ raises the value of marketable skills for married female agents without affecting their market income when they remain single or, equivalently and more importantly, their threat point in the bargaining process. This point also offers a serious policy implication, which will be discussed in section 5.3.

5.2 Welfare

We now investigate the efficient allocation of the economy in order to derive some welfare implications. The social planner's problem is fairly simple: since all terms are linear under the current setup, the efficient allocation is achieved if and only if each agent's contribution to the social welfare is maximized. First, it can immediately be observed that the planner always chooses the highest market value. Given this observation,

the planner's problem is defined as finding $e_j^{\text{opt}}(x)$, $j = f, m$, such that

$$e_f^{\text{opt}}(x) = \operatorname{argmax}_e \left((1 + \alpha)(1 - \theta) + 2\delta \right) e - C(e, x), \quad (12)$$

$$e_m^{\text{opt}}(x) = \operatorname{argmax}_e (1 + \alpha + 2\delta) e - C(e, x), \quad (13)$$

for married agents. The next proposition characterizes the efficient allocation of the economy.

Proposition 3 (Efficiency) *In the efficient allocation, agents marry if and only if $x \geq x^{\text{opt}}$ for some $x^{\text{opt}} \in (0, \bar{x})$. The efficient investment choices, denoted by $e_j^{\text{opt}}(x)$ and $q_j^{\text{opt}}(x)$, are given by*

$$q_f^{\text{opt}}(x) = q_m^{\text{opt}}(x) = 1, \\ e_f^{\text{opt}}(x)^\gamma = \left((1 + \alpha)(1 - \theta) + 2\delta \right) \frac{x}{\beta}, \quad e_m^{\text{opt}}(x)^\gamma = (1 + \alpha + 2\delta) \frac{x}{\beta},$$

for $x \geq x^{\text{opt}}$.

Proof. See Appendix.

Note that our model includes premarital investments and, therefore, has a holdup property. In the standard holdup problem, the investment level typically falls below its efficient level because concerned parties fail to take potential partners' benefits into consideration. As Peters and Siow (2002) show, however, this problem can substantially be alleviated when agents compete for spouses: with the competitive marriage market, there may arise an additional incentive to invest in order to make them more attractive for potential partners. In our model with intrahousehold bargaining, however, this competition effect influences the equilibrium allocation in different ways, depending crucially on whether Condition D holds or not.

We first consider the allocation in the asymmetric equilibrium. In the limiting case where δ approaches zero, for married male agents,

$$e_m^{\text{asym}}(x)^\gamma = \left(\frac{\alpha}{2} + 1 \right) \frac{x}{\beta}. \quad (14)$$

It is immediate to see that $e_m^{\text{asym}}(x) < e_m^{\text{opt}}(x)$, i.e., there is underinvestment. The reason for this is that in this equilibrium, married female agents make an investment whose social value is infinitesimally small when δ approaches zero. There is virtually no competition effect for male agents who invest strictly for their own sake, without regarding its effect on potential partners. In general, therefore, potential competition in the marriage market does not work to resolve this typical holdup problem when the intrinsic value of skills is sufficiently small.

While the typical holdup problem occurs for male agents, a more serious inefficiency arises at the other end for female agents who have an incentive to strategically degrade the market value of skills. In the asymmetric equilibrium, married female agents choose the lowest market value ($q_f(x) = 0$) while the level of skills is given by

$$e_f^{\text{asym}}(x)^{1+\gamma} = \frac{\alpha}{2} \left(\frac{\alpha}{2} + 1 \right)^{\frac{1}{\gamma}} \left(\frac{x}{\beta} \right)^{\frac{1+\gamma}{\gamma}} - \gamma \left(\frac{\alpha}{2} + 1 \right)^{\frac{1}{\gamma}} \left(\frac{x^*}{\beta} \right)^{\frac{1+\gamma}{\gamma}} \quad (15)$$

This investment pattern is clearly inefficient because the planner always chooses the highest productivity in the efficient allocation. Taking this inefficient choice as given, on the other hand, this can also be seen as a type of overinvestment problem because women should not choose a positive level of investment if its market value is zero. Female agents make an inefficient investment strictly to make themselves more attractive in the marriage market to secure the benefit of marriage. In this sense, it is clearly an understatement to say that the competition effect fails to resolve the holdup problem: in this case, it is actually the source of another type of inefficiency.

While the degree of inefficiency is much less severe, the competition effect is not sufficient to achieve the full efficiency even in the symmetric equilibrium when married male and female agents are asymmetric in terms of the returns to skills. Again, married female agents tend to overinvest while married male agents tend to underinvest:

$$e_f^{\text{sym}}(x)^\gamma - e_f^{\text{opt}}(x)^\gamma = \frac{\theta(1+\alpha)x}{2} \frac{1}{\beta} > 0, \quad (16)$$

$$e_m^{\text{sym}}(x)^\gamma - e_m^{\text{opt}}(x)^\gamma = \frac{-\theta(1+\alpha)x}{2} \frac{1}{\beta} < 0. \quad (17)$$

When both married female and male agents choose the highest market value, they can earn the same market income when they remain single and thus end up with the same level of utility in equilibrium. This implies that the returns to the investment are identical for female and male agents from the individual point of view: as a result, the competition effect is excessively strong for female agents. Note that this inefficiency stems strictly from the exogenously assumed asymmetry between male and female agents. As this inherent difference becomes smaller, i.e., $\theta \rightarrow 0$, the inefficiency disappears and the efficient outcome is realized as in Peters and Siow (2002).

5.3 Policy implications

When Condition D holds, the resulting equilibrium is inefficient in that female agents intentionally degrade the market value of skills even though the cost of acquiring skills is totally independent of it. Our analysis reveals that a direct subsidy to the acquisition of marketable skills is not an effective tool to deal with this inefficiency. Within the current setup, a subsidy to the acquisition of skills can be seen as a reduction in β for female agents, possibly financed by lump-sum taxes imposed on all agents. As we have already seen, however, this type of policy intervention fails to resolve the inefficiency at its origin: since a reduction in β could benefit both single and married agents proportionally and hence is not at all effective in narrowing the earnings differential, such a policy intervention has no impact on the nature of equilibrium.

By the same logic, the inception of affirmative action programs or equal employment opportunity laws is equally ineffective unless they are specifically targeted at married women. To see the effects of such policies, we modify the model so that the market income is lower for female agents by design, possibly due to some labor market discrimination. We now denote the market income as $\lambda_i q_i e_i$, $i = m, f$, where $\lambda_m \geq \lambda_f$. Moreover, to simplify notation, let $\lambda_m = 1$ and $\lambda_f = \lambda$. Given this, the bargaining outcome for male agents (5) is modified as

$$\begin{aligned} V_m &= \frac{1}{2} \left(y(h_f, h_m) - \lambda q_f e_f - q_m e_m \right) + q_m e_m. \\ &= \left(\left(\frac{\alpha}{2} + 1 \right) q_m + \delta \right) e_m + \delta a_f(q_f, e_f) - d, \end{aligned} \quad (18)$$

where a_f is now defined as

$$a_f(q_f, e_f) = \frac{1}{\delta} (\Delta \lambda q_f + \delta) e_f, \quad (19)$$

Even under this unequal treatment by gender, the preferences of male agents are qualitatively unchanged: male agents still prefer partners with less marketable skills as long as Condition D holds. As a consequence, the nature of equilibrium is almost identical to that in Proposition 1.

Proposition 4 (Asymmetric equilibrium with unequal treatment by gender) *Suppose that (i) Condition D holds and (ii) δ is sufficiently close to zero. Then, there exists an asymmetric equilibrium where agents choose to marry if and only if $x \geq x^*$. In the limiting case where $\delta \rightarrow 0$, the optimal investment*

choices in the asymmetric equilibrium, denoted by $q_j^{\text{at}}(x)$ and $e_j^{\text{at}}(x)$, are given by

$$q_f^{\text{at}}(x) = 0, \quad q_m^{\text{at}}(x) = 1,$$

$$e_f^{\text{at}}(x)^{1+\gamma} = e_f^{\text{asym}}(x)^{1+\gamma} + \gamma(1 - \lambda^{\frac{1+\gamma}{\gamma}}) \left(\frac{x^*}{\beta} \right)^{\frac{1+\gamma}{\gamma}}, \quad e_m^{\text{at}}(x) = e_m^{\text{asym}}(x).$$

Proof. See Appendix.

Any policy intervention aimed at raising the returns to skills for female agents λ , such as affirmative action programs, will reduce the amount of investment in the asymmetric equilibrium because female agents are now treated more fairly in the labor market. This certainly weakens their incentive to marry and hence the competition effect in the marriage market. As is true for the change in the overall productivity of skills β , however, any policy of this kind has no impact on the nature of equilibrium since it does not affect the earnings differential. This implies that policy interventions such as affirmative action programs may be effective in closing the gender gap in the short run where the investment choices are fixed, they are not at all effective in the long run as they fail to fundamentally resolve the inefficiency of the asymmetric equilibrium.

As an important implication of the model, there is arguably a better alternative to solve this problem. We have thus far seen that the source of the inefficiency lies in the earnings differential between single and married women. Given this, it is conceptually straightforward to find the solution. The key is to reduce θ by compensating married women for the lost market income which would have been earned if they had not had to engage in domestic activities. There is indeed a wide array of possible compensation programs to achieve this goal: for instance, monetary transfers to compensate for the opportunity cost of childbearing and child rearing through providing paid maternity leaves, child care benefits (made proportionally to beneficiaries' potential earnings), or subsidies to nursery schools can be a simple and effective policy measure in eliminating the inefficient outcome.

6 Conclusion

This paper provides a model that can account for the gender segregation of skill acquisition when the marriage market is competitive. The key ingredients of the model are the process of intrahousehold bargaining and the cost asymmetry of domestic activities. We show that when the cost of domestic activities is asymmetrically

placed on women, their investment pattern may be distorted to gain advantages in the marriage market. This leads to the emergence of the inefficient asymmetric equilibrium where a bulk of women intentionally degrade the market value of acquired skills. In the asymmetric equilibrium, there is an incentive for women to acquire the least marketable skills even when the intrinsic value of those skills is arbitrarily small.

The model indicates that the inefficient asymmetric equilibrium arises when the cost asymmetry of domestic activities is more significant (Condition D). This result offers a critical policy implication: the fundamental source of the gender segregation of skill acquisition is the earnings differential between single and married women, rather than that between men and women. Once married, women devote more resources to domestic activities in order to reap the benefit of role specialization. This lowers the returns to marketable skills when they are married, compared to when they remain single. The model indicates that an effective remedy for this is to correct the cost structure of domestic activities so that Condition D no longer holds. A policy intervention which redistributes the cost of domestic activities, which is initially concentrated more heavily on married women, to all members of the economy can be much more effective than an intervention, such as affirmative action, which directly subsidizes the acquisition of marketable skills for women.

Appendix

Proof of Proposition 1.

We first consider the problem faced by male agents. It follows from the property of the utility function (10) that $q_m = 1$ in any equilibria, since U_m is strictly increasing in q_m . By raising the market value of skills q_m , male agents become more productive in the labor market and, at the same time, more attractive in the marriage market. In the following, therefore, we impose that $q_m = 1$ and, hence, $a_m = e_m$. The utility function (10) is now simplified as follows:

$$U_m(h_m, x) = \left(\frac{\alpha}{2} + \delta + 1\right)e_m + \delta\phi^{-1}(e_m) - d - C(e_m, x). \quad (\text{A.1})$$

The problem faced by female agents is much more complicated. To characterize the optimal investment choice, we divide the maximization problem into two stages. In the first stage, female agents choose e_f and q_f to maximize U_f keeping their own attractiveness a_f constant (and, hence, keeping matching partners identical). The first-stage problem is thus formally defined as

$$\max_{e_f, q_f} \left(\frac{\alpha}{2} + \delta\right)\phi(\tilde{a}_f) + \left((\Delta + 1)q_f + \delta\right)e_f - d - C(e_f, x),$$

subject to

$$\tilde{a}_f = \frac{1}{\delta}(\Delta q_f + \delta)e_f,$$

for each given \tilde{a}_f . In the second stage, they then choose the attractiveness itself that attains the highest utility.

It is critical to note that when condition D holds, female agents face a tradeoff between q_f and e_f . The optimal investment choice depends on the required attractiveness \tilde{a}_f . As can be seen from the constraint, there is an inverse relationship between q_f and e_f for any given \tilde{a}_f : an increase in e_f directly implies a decrease in q_f with \tilde{a}_f being fixed. When \tilde{a}_f is sufficiently low, there is no need to reduce q_f since it is less costly to increase e_f (because of the increasing marginal cost). As \tilde{a}_f increases, however, it ultimately reaches a point where a further increase in e_f is simply too costly, and female agents start decreasing q_f . This tradeoff is summarized in the following lemma.

Lemma A.1 *For some given \tilde{a}_f , the optimal investment choice is given by*

$$(q_f, e_f) = \begin{cases} (0, \tilde{a}_f), & \text{when } e_f^u(x) < \tilde{a}_f, \\ \left(\delta(\tilde{a}_f - e_f^u(x))/(\Delta e_f^u(x)), e_f^u(x) \right) & \text{when } e_f^u(x) (\Delta + \delta)/\delta < \tilde{a}_f < e_f^u(x) \\ (1, \delta\tilde{a}_f/(\Delta + \delta)) & \text{when } \tilde{a}_f < e_f^u(x) (\Delta + \delta)/\delta, \end{cases}$$

where $e_f^u(x)$ solves

$$\delta = -\Delta \frac{\partial C(e_f^u(x), x)}{\partial e}.$$

Proof. Solving the constraint yields

$$q_f = \frac{\delta\tilde{a}_f}{\Delta e_f} - \frac{\delta}{\Delta}. \quad (\text{A.2})$$

Substituting this into the objective function, we can rewrite the maximization problem as follows:

$$\max_{e_f} \left(\frac{\alpha}{2} + \delta \right) \phi(\tilde{a}_f) + \frac{\Delta + 1}{\Delta} \delta \tilde{a}_f - \frac{\delta}{\Delta} e_f - d - C(e_f, x).$$

The maximization problem is now defined over a single variable e_f . Since $q_f \in [0, 1]$, e_f must satisfy

$$\tilde{a}_f \leq e_f, \quad (\Delta + \delta)e_f \leq \delta\tilde{a}_f. \quad (\text{A.3})$$

When there is no restriction on e_f , the maximum is achieved at $e_f = e_f^u(x)$ where $e_f^u(x)$ is defined by equation (A.1). To characterize the optimal solution, we need to consider two cases, $\Delta + \delta > 0$ and $\Delta + \delta < 0$, in turn.

1. $\Delta + \delta > 0$

In this case, the possible range of e_f is

$$\tilde{a}_f \leq e_f \leq \frac{\delta}{\Delta + \delta} \tilde{a}_f. \quad (\text{A.4})$$

It is clear that the optimal value of e_f is e_f^u when e_f^u happens to lie within this range. Otherwise, the maximizer is on corner. When $\tilde{a}_f > e_f^u(x)$, then it is optimal to choose $e_f = \tilde{a}_f$ (and $q_f = 0$). When $e_f^u(x) > \delta/(\Delta + \delta)\tilde{a}_f$, on the other hand, it is optimal to choose $e_f = \delta/(\Delta + \delta)\tilde{a}_f$ (and $q_f = 1$). The results are precisely as indicated in the lemma.

2. $\Delta + \delta < 0$

In this case, the constraints (A.3) can be written as

$$e_f \geq \tilde{a}_f, \quad e_f \geq \frac{\delta \tilde{a}_f}{\Delta + \delta}. \quad (\text{A.5})$$

When \tilde{a}_f is positive, the second constraint is not binding. Then, $e_f = e_f^u(x)$ (and $0 < q_f < 1$) if $\tilde{a}_f < e_f^u(x)$. Otherwise, $e_f = \tilde{a}_f$ and $q_f = 0$. When \tilde{a}_f is negative, the first constraint is not binding. Then, $e_f = e_f^u(x)$ (and $0 < q_f < 1$) if $\tilde{a}_f > e_f^u(x)(\Delta + \delta)/\delta$. Otherwise, $e_f = \delta\tilde{a}_f/(\Delta + \delta)$ and $q_f = 1$. As above, therefore, the results are as indicated in the lemma.

Q.E.D.

After determining the optimal combination of (q_f, e_f) for each a_f , agents choose the optimal level of a_f in the second stage. Note in particular that, when a_f is in the middle range (the second case in the lemma), the level of skills is held constant at $e_f^u(x)$, and the productivity q_f is adjusted to achieve the targeted attractiveness. The maximized utility in this range is then simplified by substituting the constraint and eliminating q_f :

$$U_f(a_f, x) = \left(\frac{\alpha}{2} + \delta\right)a_m + \delta\frac{\Delta + 1}{\Delta}a_f - \frac{\delta}{\Delta}e_f^u(x) - d - C(e_f^u(x), x). \quad (\text{A.6})$$

The slope of the indifference curve is thus flat, and the slope is steeper when $q_f = 0$ as Figure 1 shows. Since the slope of the return function is equal to that of the indifference curve at the optimal point, the following lemma is immediate.

Lemma A.2 *Suppose that female agents with ability x choose to marry in equilibrium. Then, $q_f(x) = 0$ and $e_f(x) = a_f(x)$ if*

$$\left(\frac{\alpha}{2} + \delta\right)\phi'(a_f(x)) > -\frac{\delta(1 + \Delta)}{\Delta}.$$

In the analysis so far, we have focused on the case where agents decide to marry. At the beginning of the second period, however, each agent can choose whether to enter the marriage market in search of a marital partner. Under the current setup, the gain from marriage is proportional to the total market income of the household. This implies that male agents with high ability have stronger incentives to marry since they also invest more in skills. On the other hand, agents with relatively low ability may choose not to marry due to the presence of the fixed cost d .

Lemma A.3 Suppose that (i) condition D holds and (ii) δ is sufficiently close to zero. Then, there exists some threshold x^* such that agents choose to marry if and only if $x \geq x^*$. In the limiting case where $\delta \rightarrow 0$, the threshold x^* is given by

$$x^{\frac{1}{\gamma}} = \frac{1+\gamma}{\gamma} \beta^{\frac{1}{\gamma}} d \left(\left[\frac{\alpha}{2} + 1 \right]^{\frac{1+\gamma}{\gamma}} - 1 \right)^{-1}.$$

Proof. To see when it is optimal to remain single, we first need to obtain the expected gain of doing so. If an agent chooses to remain single, the only source of utility is the market income. The expected utility when single (notice that the expected utility when single is independent of gender) can be written as

$$qe - C(e, x) = qe - \frac{\beta}{x(1+\gamma)} e^{1+\gamma}. \quad (\text{A.7})$$

It then directly follows from this that the optimal investment choices when single, denoted by e^s and q^s , are given by

$$\begin{aligned} e^s(x) &= \left(\frac{x}{\beta} \right)^{\frac{1}{\gamma}}, \\ q^s(x) &= 1. \end{aligned} \quad (\text{A.8})$$

Now let $S(x)$ denote the expected indirect utility when single, which is obtained as

$$\begin{aligned} S(x) &\equiv \max_{q,e} qe - C(e, x), \\ &= \frac{\gamma}{1+\gamma} \left(\frac{x}{\beta} \right)^{\frac{1}{\gamma}}. \end{aligned} \quad (\text{A.9})$$

If an agent is to marry, the expected utility when married must exceed $S(x)$.

We now examine the expected benefit of marriage in the asymmetric equilibrium when $q_f = 0$ for all married female agents. To this end, define

$$M_j(x) \equiv \max_{e_j} U_j(e_j, x).$$

as the expected value of marriage. In the asymmetric equilibrium where $e_j = a_j$, $q_f = 0$, and $q_m = 1$, the expected value can be written as

$$M_m(x) = \max_{a_m} \left(\frac{\alpha}{2} + \delta + 1 \right) a_m + \delta \phi^{-1}(a_m) - d - C(a_m, x), \quad (\text{A.10})$$

$$M_f(x) = \max_{a_f} \left(\frac{\alpha}{2} + \delta \right) \phi(a_f) + \delta a_f - d - C(a_f, x). \quad (\text{A.11})$$

Note that as $\delta \rightarrow 0$, the expected value of marriage for male agents is determined independently of a_f . This implies that the critical value x^* that solves $M_m(x^*) = S(x^*)$ is independent of the return function of ϕ .

As $\delta \rightarrow 0$, the optimal investment level $a_m(x) = e_m(x)$ and the expected benefit M_m can be obtained as follows.

$$a_m(x) = \left(\frac{\alpha}{2} + 1 \right)^{\frac{1}{\gamma}} \left(\frac{x}{\beta} \right)^{\frac{1}{\gamma}}, \quad (\text{A.12})$$

$$M_m(x) = \frac{\gamma}{1+\gamma} \left(\frac{\alpha}{2} + 1 \right)^{\frac{1+\gamma}{\gamma}} \left(\frac{x}{\beta} \right)^{\frac{1}{\gamma}} - d. \quad (\text{A.13})$$

The threshold x^* is then obtained as

$$x^* \frac{1}{\gamma} = \frac{1+\gamma}{\gamma} \beta^{\frac{1}{\gamma}} d \left(\left[\frac{\alpha}{2} + 1 \right]^{\frac{1+\gamma}{\gamma}} - 1 \right)^{-1}. \quad (\text{A.14})$$

In order to show that $M_j(x) > S(x)$ for $x > x^*$, first note that $M_j(x^*) = S(x^*)$. Thus, it is enough to show that $M_j'(x) > S'(x)$ for $x > x^*$. From the envelop theorem, we have,

$$M_j'(x) - S'(x) = -C_x(a_j(x), x) + C_x(e^s(x), x),$$

where C_x is the partial derivative with respect to the second element. Since we assume that $C_{xe} < 0$, it is clear that $M_j'(x) > S'(x)$ if $a_j(x) > e^s(x)$. For male agents, it is clear that $a_m(x) > e^s(x)$ by comparing (A.8) and (A.12). For female agents, we will derive the explicit form of a_f later in (A.21) when δ goes to zero. By using this along with Assumption 2, we have

$$\begin{aligned} a_f(x)^{1+\gamma} - e^s(x)^{1+\gamma} &= \frac{\alpha}{2} \left(\frac{\alpha}{2} + 1 \right)^{\frac{1}{\gamma}} \left(\frac{x}{\beta} \right)^{\frac{1+\gamma}{\gamma}} - \gamma \left(\frac{\alpha}{2} + 1 \right)^{\frac{1}{\gamma}} \left(\frac{x^*}{\beta} \right)^{\frac{1+\gamma}{\gamma}} - \left(\frac{x}{\beta} \right)^{\frac{1+\gamma}{\gamma}}, \\ &\geq \left(\frac{\alpha}{2} \left(\frac{\alpha}{2} + 1 \right)^{\frac{1}{\gamma}} - \gamma \left(\frac{\alpha}{2} + 1 \right)^{\frac{1}{\gamma}} - 1 \right) \left(\frac{x^*}{\beta} \right)^{\frac{1+\gamma}{\gamma}} > 0. \end{aligned} \quad (\text{A.15})$$

We thus verify that $M_f(x) > S(x)$ for all $x > x^*$. Q.E.D.

We are now ready to construct an equilibrium allocation where agents with $x \geq x^*$ choose to marry. When δ is sufficiently small, $q_f(x) = 0$ for $x \geq x^*$ by lemma A.2. We also conjecture that the resulting matching pattern is assortative, i.e., any matched agents are of the same type. Given these, we explicitly construct an equilibrium allocation when $\delta \rightarrow 0$, and then verify that it indeed constitutes an equilibrium.

The first-order condition of the problem implies that for all $x \geq x^*$,

$$\left(\frac{\alpha}{2} + 1 \right) = C_e(a_m(x), x) = \frac{\beta}{x} a_m(x)^\gamma, \quad (\text{A.16})$$

$$\frac{\alpha}{2} \phi'(a_f(x)) = C_e(a_f(x), x) = \frac{\beta}{x} a_f(x)^\gamma. \quad (\text{A.17})$$

Given these, we now characterize the optimal investment level in the asymmetric equilibrium. First, the optimal investment level for male agents is already derived in (A.12). Note, on the other hand that it is more involved to characterize the optimal investment level for female agents, which depends on the shape of the return function. We thus need to obtain an explicit solution for the return function. To this end, combining (A.16) and (A.17) yields

$$\left(\frac{\phi(a_f)}{a_f} \right)^\gamma \frac{\alpha}{2} \phi'(a_f) = \left(1 + \frac{\alpha}{2} \right).$$

This equation provides a requirement that the return function ϕ must satisfy. By using the fact that $\phi(a_f) = a_m$, we obtain the following closed-form solution:

$$\frac{\alpha}{2} \left(a_m(x)^{\gamma+1} - a_m(x^*)^{\gamma+1} \right) = \left(1 + \frac{\alpha}{2} \right) \left(a_f(x)^{1+\gamma} - a_f(x^*)^{\gamma+1} \right). \quad (\text{A.18})$$

To solve this explicitly for $a_f(x)$, we need to obtain the value of $a_f(x^*)$. Since $M_f(x^*) = S(x^*)$, $a_f(x^*)$ must solve

$$\begin{aligned} 0 &= M_f(x^*) - S(x^*) = \frac{\alpha}{2}a_m(x^*) - d - C(a_f(x^*), x^*) - S(x^*), \\ &= \frac{\alpha}{2}a_m(x^*) - (M_m(x^*) + d - S(x^*)) - C(a_f(x^*), x^*) - S(x^*). \end{aligned}$$

Substituting (A.12) and (A.13) into this, this condition can be written as

$$\frac{1}{1+\gamma} \left(\frac{\alpha}{2} - \gamma \right) \left(\frac{\alpha}{2} + 1 \right)^{\frac{1}{\gamma}} \left(\frac{x^*}{\beta} \right)^{\frac{1}{\gamma}} - C(a_f(x^*), x^*) = 0. \quad (\text{A.19})$$

Solving this yields

$$a_f(x^*)^{1+\gamma} = \left(\frac{\alpha}{2} - \gamma \right) \left(\frac{\alpha}{2} + 1 \right)^{\frac{1}{\gamma}} \left(\frac{x^*}{\beta} \right)^{\frac{1+\gamma}{\gamma}}. \quad (\text{A.20})$$

This equation defines a boundary condition which must be satisfied in equilibrium. The optimal investment level for female agents is obtained from substituting this boundary condition and (A.12) into (A.18):

$$a_f(x)^{1+\gamma} = e_f(x)^{1+\gamma} = \frac{\alpha}{2} \left(\frac{\alpha}{2} + 1 \right)^{\frac{1}{\gamma}} \left(\frac{x}{\beta} \right)^{\frac{1+\gamma}{\gamma}} - \gamma \left(\frac{\alpha}{2} + 1 \right)^{\frac{1}{\gamma}} \left(\frac{x^*}{\beta} \right)^{\frac{1+\gamma}{\gamma}}. \quad (\text{A.21})$$

Finally, we will show that no agents have incentives to unilaterally change their investment choices. First, married agents cannot be made better off by remaining single, since $M_j(x) > S(x)$ for all $x > x^*$, as we have established in the proof of Lemma A.3. Second, married agents have no incentives to change the investment levels a_j . In order to prove this, redefine the maximization problems (A.10) and (A.11) as we did in (A.1) and (A.6):

$$M_j(x) = \max_{a_j} U_j(a_j, x).$$

Then, it suffices to show that $\partial U_j(\hat{a}_j, x) / \partial a_j$ is negative (positive, respectively) when $\hat{a}_j > a_j(x)$ ($\hat{a}_j < a_j(x)$). We first consider the case in which $\hat{a}_j > a_j(x)$. In this case, we can find some agents with \hat{x} who choose \hat{a}_j and, therefore, $\hat{a}_j = a_j(\hat{x})$. From the stable matching condition, $\hat{x} > x$, and, then, the optimal condition implies,

$$0 = \frac{\partial}{\partial a_j} U_j(\hat{a}_j, \hat{x}) = \frac{\partial}{\partial a_j} U_j(\hat{a}_j, x) - C_e(\hat{a}_j, \hat{x}) + C_c(\hat{a}_j, x) > \frac{\partial}{\partial a_j} U_j(\hat{a}_j, x).$$

The last inequality follows from the fact that $C_{ax}(a_j, x) < 0$. We can obtain the opposite sign when $\hat{a}_j < a_j(x)$. We thus prove that this allocation indeed constitutes an equilibrium when $\delta \rightarrow 0$. Q.E.D.

Proof of Proposition 2.

To prove the proposition, we first establish the following result.

Lemma A.4 Suppose that (i) Condition D does not hold and (ii) $e = \psi(e)$ for all matched pairs. Then, there exists some threshold x^{**} such that agents choose to marry if and only if $x \in [x^{**}, \bar{x}]$ where the threshold is given by

$$x^{**\frac{1}{\gamma}} = \frac{1+\gamma}{\gamma} \beta^{\frac{1}{\gamma}} d \left(\left[1 + 2\delta + \frac{\alpha}{2} + \frac{\alpha - \theta(1+\alpha)}{2} \right]^{\frac{1+\gamma}{\gamma}} - 1 \right)^{-1}.$$

Proof. Given that $q_j(x) = 1$ for all x , the indirect utility when married is obtained as follows:

$$M_f(x) = \max_e \left(\frac{\alpha}{2} + \delta \right) (e + \psi(e)) + e - \frac{\theta(1+\alpha)}{2} e - d - C(e, x), \quad (\text{A.22})$$

$$M_m(x) = \max_e \left(\frac{\alpha}{2} + \delta \right) (e + \psi^{-1}(e)) + e - \frac{\theta(1+\alpha)}{2} \psi^{-1}(e) - d - C(e, x). \quad (\text{A.23})$$

Note that if $e = \psi(e)$, these are reduced to

$$M_j(x) = \max_e \left(1 + 2\delta + \alpha - \frac{\theta(1+\alpha)}{2} \right) e - d - C(e, x). \quad (\text{A.24})$$

The first-order condition then implies that

$$e^\gamma = \left(1 + 2\delta + \alpha - \frac{\theta(1+\alpha)}{2} \right) \frac{x}{\beta}, \quad (\text{A.25})$$

which leads to

$$M_j(x) = \frac{\gamma}{1+\gamma} \left(1 + 2\delta + \alpha - \frac{\theta(1+\alpha)}{2} \right)^{\frac{1+\gamma}{\gamma}} \left(\frac{x}{\beta} \right)^{\frac{1}{\gamma}} - d. \quad (\text{A.26})$$

The threshold x^{**} must satisfy $M_j(x^{**}) = S(x^{**})$, from which we obtain

$$\left(\frac{x^{**}}{\beta} \right)^{\frac{1}{\gamma}} = \frac{1+\gamma}{\gamma} d \left(\left[1 + 2\delta + \alpha - \frac{1}{2}\theta(1+\alpha) \right]^{\frac{1+\gamma}{\gamma}} - 1 \right)^{-1}. \quad (\text{A.27})$$

It is clear that $M_j(x) > S(x)$ for all $x > x^{**}$ when condition D does not hold. Q.E.D.

Given this proof, it is straightforward to verify that the allocation derived in lemma A.4 indeed constitutes an equilibrium. Q.E.D.

Proof of Proposition 3.

By maximizing the objective functions (12) and (13), we can obtain the socially efficient investment levels for married agents:

$$h_f^{\text{opt}}(x)^\gamma = \left((1+\alpha)(1-\theta) + 2\delta \right) \frac{x}{\beta}, \quad h_m^{\text{opt}}(x)^\gamma = (1+\alpha + 2\delta) \frac{x}{\beta}.$$

We can then obtain the expected value of marriage for each gender subset:

$$M_f(x) = \frac{\gamma}{1+\gamma} \left((1+\alpha)(1-\theta) + 2\delta \right)^{\frac{1+\gamma}{\gamma}} \left(\frac{x}{\beta} \right)^{\frac{1}{\gamma}} - d, \quad M_m(x) = \frac{\gamma}{1+\gamma} (1+\alpha + 2\delta)^{\frac{1+\gamma}{\gamma}} \left(\frac{x}{\beta} \right)^{\frac{1}{\gamma}} - d.$$

Since the gain from marriage is stronger for agents with high ability, the efficient allocation requires that the matching pattern be positively assortative. At the threshold x^{opt} , the total benefit $M_f(x^{\text{opt}}) + M_m(x^{\text{opt}})$ must equal to the benefit when single $2S(x^{\text{opt}})$, i.e.,

$$\begin{aligned} 0 &= M_f(x^{\text{opt}}) + M_m(x^{\text{opt}}) - 2S(x^{\text{opt}}), \\ &= \frac{\gamma}{1+\gamma} \left(\frac{x^{\text{opt}}}{\beta} \right)^{\frac{1}{\gamma}} \left[\left((1+\alpha)(1-\theta) + 2\delta \right)^{\frac{1+\gamma}{\gamma}} + (1+\alpha+2\delta)^{\frac{1+\gamma}{\gamma}} - 2 \right] - 2d. \end{aligned}$$

Solving this equation yields the following threshold value:

$$x^{\text{opt} \frac{1}{\gamma}} = \frac{1+\gamma}{\gamma} \beta^{\frac{1}{\gamma}} 2d \left[\left((1+\alpha)(1-\theta) + 2\delta \right)^{\frac{1+\gamma}{\gamma}} + (1+\alpha+2\delta)^{\frac{1+\gamma}{\gamma}} - 2 \right]^{-1}.$$

It is also clear that $M_f(x) + M_m(x) > 2S(x)$ for all $x > x^{\text{opt}}$. Q.E.D.

Proof of Proposition 4.

Define a variable q'_f such that $q'_f = \lambda q_f$. By replacing q_f with this new variable q'_f , the problem is essentially unchanged except for the fact that the range of q'_f is $[0, \lambda]$. The results regarding the asymmetric equilibrium thus remain true, since $q_f(x) = 0$ for married female agents and λ does not affect the solution as long as female agents choose to marry. The market income when single, on the other hand, is affected by this modification since single female agents choose $q_f(x) = 1$ (or $q'_f(x) = \lambda$). The optimal investment choice for female agents when single is now modified as

$$e_f^s(x) = \left(\frac{\lambda x}{\beta} \right)^{\frac{1}{\gamma}}. \quad (\text{A.28})$$

Because of unequal treatment in the labor market, the indirect utility when single (A.9) is now dependent on gender.

We now denote the indirect utility for female agents when single as $S_f(x)$, which is given by

$$\begin{aligned} S_f(x) &\equiv \max_{e_f} \lambda e_f - \frac{1}{x(1+\gamma)} \beta e_f^{1+\gamma}, \\ &= \frac{\lambda \gamma}{1+\gamma} \left(\frac{\lambda x}{\beta} \right)^{\frac{1}{\gamma}}. \end{aligned} \quad (\text{A.29})$$

With this modification, we can show that all of the results, including Lemma A.2 and A.3, remain true.

We can also show that the optimal investment choice for male agents is determined independently of λ . In the limiting case, any discriminatory act in the labor market has no effect on male agents' behavior in the asymmetric equilibrium. As a result, the optimal investment choices for male agents as well as the threshold $x = x^*$ remain the same as in the basic model. Suppose that we denote the optimal investment choices under this unequal treatment by $e_j^{\text{at}}(x)$ and $q_j^{\text{at}}(x)$. Then, our argument implies that $e_m^{\text{at}}(x) = e_m^{\text{asym}}(x)$ for all x .

We now turn our attention to female agents who face a slightly different situation. First, by definition,

$$\begin{aligned}
0 &= M_f(x^*) - S_f(x^*) \\
&= \frac{\alpha}{2} e_m^{\text{at}}(x^*) - d - C(a_f^{\text{at}}(x^*), x^*) - S_f(x^*), \\
&= \frac{\alpha}{2} e_m^{\text{asym}}(x^*) - (M_m^{\text{asym}}(x^*) + d - S(x^*)) - C(a_f^{\text{at}}(x^*), x^*) - S_f(x^*), \\
&= \frac{\alpha}{2} \left(\frac{\alpha}{2} + 1 \right)^{\frac{1}{\gamma}} \left(\frac{x^*}{\beta} \right)^{\frac{1}{\gamma}} - \frac{\gamma}{1+\gamma} \left(\frac{\alpha}{2} + 1 \right)^{\frac{1+\gamma}{\gamma}} \left(\frac{x^*}{\beta} \right)^{\frac{1}{\gamma}} - C(a_f^{\text{at}}(x^*), x^*) + S(x^*) - S_f(x^*), \\
&= \frac{1}{1+\gamma} \left(\frac{\alpha}{2} - \gamma \right) \left(\frac{\alpha}{2} + 1 \right)^{\frac{1}{\gamma}} \left(\frac{x^*}{\beta} \right)^{\frac{1}{\gamma}} - \frac{\beta}{(1+\gamma)x^*} a_f^{\text{at}}(x^*)^{1+\gamma} + \frac{\gamma}{1+\gamma} \left(1 - \lambda^{\frac{1+\gamma}{\gamma}} \right) \left(\frac{x^*}{\beta} \right)^{\frac{1}{\gamma}}.
\end{aligned}$$

This yields the boundary condition $a_f^{\text{at}}(x^*)$:

$$a_f^{\text{at}}(x^*)^{1+\gamma} = a_f^{\text{asym}}(x^*)^{1+\gamma} + \gamma \left(1 - \lambda^{\frac{1+\gamma}{\gamma}} \right) \left(\frac{x^*}{\beta} \right)^{\frac{1+\gamma}{\gamma}}. \quad (\text{A.30})$$

To obtain $a_f^{\text{at}}(x)$ for $x > x^*$, we can resort to (A.18), which is clearly valid under this modification:

$$\frac{\alpha}{2} \left(a_m(x)^{1+\gamma} - a_m(x^*)^{1+\gamma} \right) = \left(1 + \frac{\alpha}{2} \right) \left(a_f(x)^{1+\gamma} - a_f(x^*)^{1+\gamma} \right).$$

This indicates that the following relationship must hold between a_f^{at} and a_f^{asym} :

$$a_f^{\text{at}}(x)^{1+\gamma} - a_f^{\text{at}}(x^*)^{1+\gamma} = a_f^{\text{asym}}(x)^{1+\gamma} - a_f^{\text{asym}}(x^*)^{1+\gamma}.$$

By substituting (A.30) into the above equation, we can show that

$$a_f^{\text{at}}(x)^{1+\gamma} = a_f^{\text{asym}}(x)^{1+\gamma} + \gamma \left(1 - \lambda^{\frac{1+\gamma}{\gamma}} \right) \left(\frac{x^*}{\beta} \right)^{\frac{1+\gamma}{\gamma}}.$$

Q.E.D.

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Table 1. Percentage of tertiary qualifications awarded to females (2002)

| | Tertiary A programs | Advanced research | Social sciences | Engineering |
|-----------------|---------------------|-------------------|-----------------|-------------|
| Iceland | 66 | 40 | 59 | 27 |
| Denmark | 66 | 41 | 45 | 23 |
| Norway | 63 | 37 | 48 | 22 |
| Finland | 63 | 48 | 68 | 21 |
| Poland | 63 | 44 | 67 | 24 |
| New Zealand | 62 | 47 | 57 | 32 |
| Hungary | 62 | 45 | 58 | 26 |
| Sweden | 61 | 41 | 59 | 28 |
| Spain | 59 | 45 | 60 | 29 |
| Ireland | 59 | 40 | 58 | 22 |
| France | 58 | 43 | 60 | 25 |
| United States | 57 | 46 | 54 | 22 |
| Greece | 57 | 38 | n.a. | n.a. |
| Australia | 57 | 44 | 53 | 23 |
| Italy | 57 | 52 | 55 | 28 |
| United Kingdom | 56 | 42 | 55 | 20 |
| Netherlands | 55 | 38 | 50 | 13 |
| Slovak Republic | 55 | 41 | 55 | 31 |
| Mexico | 53 | 39 | 57 | 25 |
| Czech Republic | 53 | 34 | 55 | 23 |
| Belgium | 51 | 36 | 54 | 21 |
| Austria | 49 | 38 | 51 | 17 |
| Germany | 49 | 36 | 45 | 21 |
| Korea | 48 | 23 | 42 | 25 |
| Switzerland | 44 | 34 | 37 | 14 |
| Turkey | 41 | 34 | 39 | 23 |
| Japan | 39 | 23 | 33 | 10 |
| Country mean | 55 | 40 | 53 | 23 |

Source: OECD (2004), Table A4.2.

Note: Tertiary A programs are largely theory based and typically last four or more years. Advanced research programs lead directly to the award of an advanced research qualification such as Ph.D. Social Sciences include social sciences, business, law and services. Engineering includes engineering, manufacturing, and construction.

Figure 1. The optimal choice of a_j

