Farsighted Stability in Two-Person Strategic Form Games

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Extended Abstract

The purpose of this research is to study farsighted stability proposed by Chwe(1994) in two-person strategic form games.

We employ two notions of farsighted stability proposed by Chwe (1994): the farsighted stable set and the largest consistent set. More precisely, we allow coalitional moves of players, and study the farsighted stable set and the largest consistent set of coalitional contingent threat situation. Refer to Chwe (1994) and Greenberg (1990).

The stable set was originally defined by von Neumann and Morgenstern (1953). Though it was defined as a solution for characteristic function form cooperative games, von Neumann and Morgenstern explained the notion of stable set in more abstract games. Later Greenberg (1990) cleared the way in applying this notion to noncooperative strategic form games.

The stable set is a set of outcomes satisfying two stability notions: internal stability and external stability. Internal stability implies that between any two outcomes in the set, there is no group of individuals whose members all prefer one to the other and can achieve the preferred outcome. External stability implies that for any outcome outside the set, there is a group of individuals whose members all have a commonly preferred outcome in the set and can realize it. Both von Neumann/Morgenstern and Greenberg assume a single move by a group of individuals in defining the two types of stability.

Chwe criticized the above for being too myopic. After a group of players move, another group may move, and then a third group of players may move, and so on. Thus, the group of players moving first must take into account such a sequence of moves and evaluate their profits in the end. A similar idea was proposed by Harsanyi (1974), albeit in a cooperative game theory context.

Chwe incorporated sequences of moves into the definition of a stable set. We call such a set a farsighted stable set. He defined another solution as a consistent set. The consistent set is very similar to the farsighted stable set but differs from it in how deviating individuals expect moves that may follow. In the case of a farsighted stable set, defectors are optimistic and expect that a sequence of moves most favorable to them would follow. On the contrary, the consistent set assumes that defectors have pessimistic expectation. The consistent set that contains all other consistent sets is called the largest consistent set.

It will be shown that the following holds in two-person starategic form games.

1. General two-person games
   (a) If a strategy combination is strictly enforceable and strongly Pareto efficient, then the set of strategy combinations with payoffs equal to it is a farsighted stable set.

(b) Individually irrational outcomes are contained neither in any farsighted stable set nor in the largest consistent set.

(c) In two-person zero-sum games, if the maximin and the minimax values are equal, then the set of outcomes which gives one player the value of the game is the unique farsighted stable set; and further the set is also the largest consistent set.

2. Prisoner's dilemma

(a) When players use only pure strategies, (Cooperate, Cooperate) is the unique farsighted stable set and also the largest consistent set.

(b) In the mixed extension, farsighted stable sets are essentially singleton sets, each consisting of a Pareto efficient and strictly individually rational strategy combinations, while the largest consistent set consists of all individually rational strategy combinations.

3. Duopoly markets

(a) In Cournot duopoly, the unique farsighted stable set consists of combinations of production levels which maximize their joint profits. The largest consistent set consists of most of possible combinations of production levels.

(b) In Bertrand duopoly, the unique farsighted stable set consists of combination of monopoly prices. The largest consistent set consists of most of possible combinations of price levels.

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