Debreu's Social Equilibrium Theorem with Asymmetric Information and a Continuum of Agents*

by

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Abstract: We provide several different generalizations of Debreu's social equilibrium theorem by allowing for asymmetric information and a continuum of agents. The results not only extend the ones in Kim-Yannelis (1997), but also new theorems are obtained which allow for a convexifying effect on aggregation. The latter is achieved by imposing the assumption of "many more agents than strategies" (Rustichini-Yannelis (1991) and Tourky-Yannelis (2001)).

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1 Introduction

Debreu (1952) introduced the idea of a social system (or abstract economy a generalization of Nash's [1950, 1951] normal form game)) and proved the existence of an equilibrium for a social system. This result was the main mathematical tool used to prove the existence of a Walrasian equilibrium for a concrete economy by Arrow and Debreu (1954).

The purpose of this paper is twofold. First, to generalize the idea of a social system by introducing asymmetric information, and a continuum of agents. Second, to prove several existence of an equilibrium theorems for a social system with asymmetric information and with a continuum of agents.

Our first theorem extends several known results. First, it extends the Schmeidler (1971) theorem to allow for asymmetric information. Second, it extends the Kim-Yannelis (1997) and Balder (2001) existence results from a Bayesian normal form game with a continuum of players to a social Bayesian system with a continuum of agents. Furthermore, it extends the Bayesian social equilibrium existence theorem of Yannelis (2002) from a finite set of agents to a continuum one.

Our second theorem is similar in spirit with our first one, but it doesn't require the assumption of concavity of the utility function. However, the externalities, i.e., the strategies of all other player affecting the utility of a player, are modeled as an integral rather than as a product, as it is the case in our first theorem. This approach allows for a convexifying effect. For abstract economies or social systems without asymmetric information this type of convexifying effect was introduced by Haller (1993), (see also Da-Rocha and Topuzu (2005)). It should be noted, that since we work with an infinite dimensional strategy space the Lyapunov theorem fails. Thus, we impose the assumption of "many more agents than strategies" which assures that the Lyapunov theorem still holds (see Rustichini-Yannelis (1991) and Tourky-Yannelis (2001) for a detailed analysis of the idea of many more agents than commodities). This assumption is obviously satisfied whenever the strategy space is finite dimensional and there is a continuum of agents.

The first two theorems are proved for a Bayesian decision making framework, i.e., agents maximize interim expected utility and update their priors. We also provide counterparts of the first two theorems for the case of ex ante expected utility maximizing behaviour. Finally, we provide a purification result, i.e., the strategy sets need not be convex and the equilibrium theorem is obtained in terms of the extreme points of the compact, convex set of strategies.

It should be mentioned that recently Balder (2004) and Cornet-Topuzu (forthcoming) have extended the Aumann (1964) Walrasian equilibrium existence theorem to allow for interdependent preferences. Our social equilibrium existence theorems with asymmetric information could be used to extend the recent deterministic results of Balder and Cornet-Topuzu by allowing for asymmetric information.

The mathematics used in this paper is rather diverse. We employ several results on the continuity and measurability of the set of integrable selections from a Banach-valued correspondence as well as results on the semicontinuity of the integral of a Banach-valued correspondence. Furthermore, some compactness and convexity results for the integral of Banach-valued correspondence play an important role.

The paper is organized as follows: The next section contains definitions. Section 3 introduces the model, i.e., the social system with asymmetric information and a continuum of agents. Section 4 contains existence results for interim expected utility functions (Bayesian decision making) and Section 5 contains similar results for ex ante expected utility function. Finally, Section 6 contains a purification result and some concluding remarks and open questions are in Section 7. Several mathematical results used in the paper are collected in the Appendix for the convenience of the reader.

2 Definitions

Let X and Y be sets. 2^A denotes the set of all nonempty subsets of the set A. $\mathcal{R}$ denotes the set of real numbers. The *graph* G_ϕ of a correspondence $\phi : X \to 2^Y$ is the set $G_\phi = \{(x, y) \in X \times Y : y \in \phi(x)\}$. If X and Y are topological spaces, $\phi : X \to 2^Y$ is said to be *lower-semicontinuous* (l.s.c) if the set $\{x \in X : \phi(x) \cap V \neq \emptyset\}$ is open in X for every open subset V of Y; $\phi : X \to 2^Y$ is said to be *upper-semicontinuous* (u.s.c) if the set $\{x \in X : \phi(x) \subset V\}$ is open in X for every open subset V of Y; $\phi : X \to 2^Y$ is said to be *continuous* if it is u.s.c. and l.s.c.

If (X, α) and (Y, β) are measurable spaces and $\phi : X \to 2^Y$ is a correspondence, ϕ is said to have a *measurable graph* if G_ϕ belongs to the product σ-algebra $a \otimes \beta$. We are often interested in the situation where (X, α) is a measurable space, Y is a topological space and $\beta = \beta(Y)$ is the

Borel σ-algebra of Y. For a correspondence ϕ from a measurable space into a topological space, if we say that ϕ has a measurable graph, it is understood that the topological space is endowed with its Borel σ-algebra. In the same setting as above, i.e., (X, α), a measurable space and Y a topological space, ϕ is said to be *lower measurable* if $\{x \in X : \phi(x) \cap V \neq \emptyset\} \in \alpha$ for every V open in Y. It follows from the projection theorem that if a correspondence has a measurable graph then it is also lower measurable. The reverse is also true if the correspondence is closed valued.

Let (T, τ, μ) be a finite measure space and X be a Banach space. For $1 \leq p < \infty$, we denote by $L_p(\mu, X)$ the space of equivalence classes of X-valued Bochner integrable functions $x : T \rightarrow X$ normed by

$$\| x \|_p = (\int_T \| x(t) \|_p \, d\mu(t))^{1/p}.$$

It is a standard result that normed by the functional $\| \cdot \|_p$ above, $L_p(\mu, X)$ becomes a Banach space [see Diestel-Uhl (1977), p.50]. We denote by S^p_ϕ the *set of all selections from* $\phi : T \rightarrow 2^X$ that belong to the space $L_p(\mu, X)$, i.e.,

$$S^p_\phi = \{x \in L_p(\mu, X) : x(t) \in \phi(t) \;\; \mu - a.e.\}.$$

We will also consider the set $S^1_\phi = \{x \in L_1(\mu, X) : x(t) \in \phi(t) \;\; \mu - a.e.\}$, i.e., S^1_ϕ is the *set of all Bochner integrable selections from* $\phi(\cdot)$. Recall that the correspondence $\phi : T \rightarrow 2^X$ is said to be *integrably bounded* if there exists a map $h \in L_1(\mu, \Re)$ such that

$\sup\{\| x \| : x \in \phi(t)\} \leq h(t) \;\; \mu - a.e.$ Moreover, note that if T is a complete measure space, X is a separable Banach space and $\phi : T \rightarrow 2^X$ is an integrably bounded, nonempty valued correspondence having a measurable graph, then by virtue of the Aumann measurable selection theorem (see Appendix) we can conclude that S^1_ϕ is nonempty. It should be noted that for the applicability of the Aumann measurable selection theorem, the range of the correspondence φ, i.e., the space X must be separable and metrizable. For this reason in the subsequent sections we will make sure that $L_1(\mu, X)$ is *separable*. To this end, we will have to assume that the measure space (T, τ, μ) and the Banach space X are both separable. This fact is used in several steps of the proofs of our theorems.

The *integral of the correspondence* $\varphi : T \to 2^X$ is defined as follows:

$$\int_T \varphi(t) d\mu(t) = \{\int_T x(t) d\mu(t) : x \in S^1_\varphi\}.$$

In the sequel we will denote the above integral by $\int_T \varphi$, or $\int \varphi$.

Observe that, if S^1_φ is nonempty, we can conclude that $\int_T \varphi$ is also nonempty.

A Banach space Y has the *Radon-Nikodym Property* (RNP) with respect to the measure space (T, τ, ν) if for each ν-continuous vector measure $G : \tau \to Y$ of bounded variation, there exists some $g \in L_1(\nu, Y)$ such that for all $E \in \tau$,

$$G(E) = \int_E g(t) d\nu(t).$$

It is a standard result (Diestel and Uhl [9]) that if Y^* (the norm dual for of Y) has the RNP with respect to (T, τ, ν), then

$$(L_1(\nu, Y))^* = L_\infty(\nu, Y^*).$$

3 The Model

3.1 The Debreu Social System with Asymmetric Information

Let $(\Omega, \mathcal{F}, \mu)$ be a complete, finite separable measure space, where Ω, denotes the set of *states of nature of the world* and the σ-algebra $\mathcal{F}$ denotes the set of *events*. Let Y be a separable Banach space whose dual has the RNP, denoting the *commodity* or *strategy* space. A *social system with asymmetric information* and with a measure[1] *space of agents* (T, τ, ν), is a set $\Gamma = \{(X, u, A, \mathcal{F}_t, q_t) : t \in T\}$ where,

1. $X : T \times \Omega \to 2^Y$ is the *random action* (*strategy*) set-valued function, where, $X(t, \omega) \subset Y$ is interpreted as the strategy set of agent t of the state of nature ω.

2. For each fixed $(t, \omega) \in T \times \Omega, u(t, \omega, \cdot, \cdot) : L_1(\nu, Y) \times X(t, \omega) \to \mathcal{R}$ is the *random utility function*, where $u(t, \omega, x, x_t)$ is interpreted as the utility function of agent t, at the state of nature ω, using his/her strategy x_t and all other players use the joint strategy x.

3. $A : T \times \Omega \times L_1(\nu, Y) \to 2^Y$, is the *random constraint correspondence* of agent t, where for all $(t, \omega, x) \in T \times \Omega \times L_1(\nu, Y)$, $A(t, \omega, x) \subset X(t, \omega)$, and $A(t, \omega, x)$ is interpreted as the constraint of agent t, when the state is ω and other agents use the joint strategy x.

4. $\mathcal{F}_t$ is the sub σ-algebra of $\mathcal{F}$ which denotes the *private information* of agent t.

5. $q_t : \Omega \to \mathcal{R}_{++}$ is the *prior* of agent t, i.e., a Radon-Nikodym derivative such that $\int_\Omega q_t(\omega) d\mu(\omega) = 1$.

[1] The measure space (T, τ, ν) is assumed to be complete, finite and separable.

Define the set $S^1_{X_t}$ as:

$$S^1_{X_t} = \{y(t) \in L_1(\mu, Y) : y(t, \cdot) : \Omega \to Y \text{ is}$$

$$\mathcal{F}_t\text{-measurable and } y(t,\omega) \in X(t,\omega) \;\; \mu - a.e.\}.$$

Notice that $S^1_{X_t}$ is the set of all Bochner integrable and $\mathcal{F}_t$-measurable selections from the random strategy set of agent t. In essence this is the set, out of which agent t will pick his/her optimal choices. In particular, an element x_t in $S^1_{X_t}$ is called a *strategy* for agent t. The typical element of $S^1_{X_t}$ is denoted by $\tilde{x}_t$ and that of $X(t,\omega)$ by $x_t(\omega)$ (or x_t). Let $S^1_X = \{\tilde{x} \in L_1(\nu, L_1(\mu, Y)) : \tilde{x}(t) \in S^1_{X_t} \;\; \nu - a.e.\}$. An element of S^1_X will be a *joint strategy profile.*

3.2 Interim Expected Utility (Bayesian case)

It will be convenient to assume throughout the paper that Ω is a *countable* set and that the σ-algebra $\mathcal{F}_t$ is generated by a countable partition Λ of Ω. For each $\omega \in \Omega$, let $E_t(\omega)$ in Λ denote the smallest set in $\mathcal{F}_t$ containing ω and to assume that for each t, $\int_{\omega' \in E_t(\omega)} q_t(\omega') d\mu(\omega') > 0$. For each fixed $(t,\omega) \in T \times \Omega$, the *interim expected utility* of agent $t, U(t,\omega,\cdot,\cdot) : S^1_X \times X(t,\omega) \to \mathcal{R}$ is defined as

$$U(t,\omega,\tilde{x},x_t) = \int_{\omega' \in E_t(\omega)} u(t,\omega',\tilde{x}(\omega'),x_t(\omega')) q_t(\omega'|E_t(\omega)) d\mu(\omega')$$

where

$$q_t(\omega'|E_t(\omega)) = \begin{cases} 0 & \text{if } \omega' \notin E_t(\omega) \\ \frac{q_t(\omega')}{\int_{\tilde{\omega} \in E_t(\omega)} q_t(\tilde{\omega}) d\mu(\tilde{\omega})} & \text{if } \omega' \in E_t(\omega). \end{cases}$$

Definition 4.1: A *social equilibrium* for Γ is a strategy profile $\tilde{x} \in S^1_X$ such that for $\nu - a.e.$

(i) $\tilde{x}(t, \omega) \in A(t, \omega, \tilde{x})$ $\mu - a.e.$ and

(ii) $U(t, \omega, \tilde{x}, \tilde{x}(t, \omega)) = \max_{y \in A(t,\omega,\tilde{x})} U(t, \omega, \tilde{x}, y)$ $\mu - a.e.$

A couple of comments are in order: First notice that $\tilde{x} \in S^1_X$ implies that for each fixed $t \in T, x(t, \cdot)$ is an $\mathcal{F}_t$-measurable selection from $X(t, \cdot)$, i.e., $x(t, \cdot)$ is chosen by agent t so that it reflects his/her own private information. Condition (i) indicates that the optimal choice is in the constraint correspondence for almost all agents and almost all states, (i.e., $\nu - a.e.$ and $\mu - a.e.$). Condition (ii) is the best reply notion, i.e., no player can deviate from his/her optimal strategy (in his/her constraint set) and increase his/her payoff once all other agents have chosen the optimal strategy vector $\tilde{x}$. Again this holds for almost all states and almost all agents, i.e., $\mu - a.e.$ and $\nu - a.e.$

4 Bayesian Social Equilibrium Existence Theorems

4.1 Existence of Social Asymmetric Equilibrium

We begin by stating the assumptions needed to prove our first existence theorem.

Assumptions

(A.1)

(a) $X : T \times \Omega \to 2^Y$ is a nonempty, convex, compact valued and integrably bounded, set valued function,

(b) For each fixed $t \in T$, $X(t, \cdot)$ has an $\mathcal{F}_t$-measurable graph, i.e., $G_{X(t,\cdot)} \in \mathcal{F}_t \otimes \mathcal{B}(Y)$.

(A.2)

(a) For each $(t, \omega) \in T \times \Omega, u(t, \omega, \cdot, \cdot) : L_1(\nu, Y) \times X(t, \omega) \to \mathcal{R}$ is continuous where $L_1(\nu, Y)$ is endowed with the weak topology and $X(t, \omega)$ with the norm topology.

(b) For each fixed $(x, y) \in L_1(\nu, Y) \times Y$, $u(\cdot, \cdot, x, y) : T \times \Omega \to \mathcal{R}$ is a measurable function.

(c) For each $(t, \omega, x) \in T \times \Omega \times L_1(\nu, Y)$, $u(t, \omega, x, \cdot) : X(t, \omega) \to \mathcal{R}$ is concave.

(d) For each $t \in T, u(t, \cdot, \cdot, \cdot)$ is integrably bounded.

(A.3)

(a) $A : T \times \Omega \times L_1(\nu, Y) \to 2^Y$ has a measurable graph.

(b) For each $(t, \omega) \in T \times \Omega, A(t, \omega, \cdot) : L_1(\nu, Y) \to 2^Y$ is a weakly continuous correspondence with closed, convex and nonempty values.

(A.4)

The correspondence $t \mapsto S^1_{X_t}$ has a measurable graph.

Theorem 1: Let Γ be a social system with asymmetric information satisfying (A.1) - (A.4). Then a social equilibrium exists in Γ.

Proof: First obeserve that the set $S^1_{X_t}$ is nonempty. Indeed in view of assumption A.1(b), by virtue of the Aumann measurable selection theorem, we can conclude that $S^1_{X_t}$ is nonempty, and similarly, we can also conclude that S_X^1 is also nonempty. It follows from (A.2) and the fact that Ω is a countable set that for each $(t,\omega) \in T \times \Omega$, $U(t,\omega,\cdot,\cdot) : S_X^1 \times X(t,\omega) \to \mathcal{R}$ is continuous[2] where S_X^1 and $L_1(\mu, Y)$ are endowed with the weak topology and $X(t,\omega)$, with the norm topology. Notice that for each fixed $(t,\tilde{x},y) \in T \times S_X^1 \times Y, U(t,\cdot,\tilde{x},y) : \Omega \to \mathcal{R}$ is $\mathcal{F}_t$-measurable and for each $(\tilde{x},y) \in S_X^1 \times Y$, $U(\cdot,\cdot,\tilde{x},y) : T \times \Omega \to \mathcal{R}$ is $\tau \otimes \mathcal{F}_t$-measurable. Furthermore, by (A.2) (c) it follows that for $(t,\omega,\tilde{x}) \in T \times \Omega \times S_X^1, U(t,\omega,\tilde{x},\cdot)$ is concave. Define the set-valued function $\bar{A} : T \times \Omega \times S_X^1 \to 2^Y$ by $\bar{A}(t,\omega,\tilde{x}) = A(t,\omega,x)$, (recall that for all $(t,\omega,x) \in T \times \Omega \times L_1(\nu, Y), A(t,\omega,x) \subset X(t,\omega)$). Define $F : T \times \Omega \times S_X^1 \to 2^Y$ by

$$F(t,\omega,\tilde{x}) = \{y \in \bar{A}(t,\omega,\tilde{x}) : U(t,\omega,\tilde{x},y) = \max_{z \in \bar{A}(t,\omega,\tilde{x})} U(t,\omega,\tilde{x},z)\}.$$

It can be easily checked that F is nonempty valued. Indeed, since all the values of the set valued function A are contained in the weakly compact set $X(\cdot,\cdot)$ and A is closed and convex (hence weakly closed), it follows that $\bar{A}$ is weakly compact valued. Since, for each $(t,\omega,\tilde{x}) \in T \times \Omega \times S_X^1$, $U(t,\omega,\tilde{x},\cdot)$ is norm u.s.c. and concave in $X(t,\omega)$, it is also weakly u.s.c. (see Balder-Yannelis [1993, Theorem 2.8]) and thus, and we conclude that F is nonempty valued. It follows from (A.2) (c), that F is convex valued.

By virtue of the Berge maximum theorem, for each fixed $(t,\omega) \in T \times \Omega, F(t,\omega,\cdot) : S_X^1 \to 2^Y$ is weakly u.s.c. Furthermore, from Castaing-Valadier [Lemma III.39, p.86, 1977], $F(t,\cdot,\tilde{x}) : \Omega \to 2^Y$ has an $\mathcal{F}_t$-measurable graph, and $F(\cdot,\cdot,\tilde{x}) : T \times \Omega \to 2^Y$ has a $\tau \otimes \mathcal{F}_t$-measurable graph.

[2]See for example Kim-Yannelis (1997, Lemma A.2) or Balder-Yannelis (1993) for complete arguments of the continuity of the expected utility.

Define the set-valued function $\varphi : T \times S^1_X \to 2^{L_1(\mu,Y)}$ by

$$\varphi(t, \tilde{x}) = \{\tilde{y}(t) \in L_1(\mu, Y) : \tilde{y}(t, \omega) \in F(t, \omega, \tilde{x}) \;\; \mu - a.e.\} \cap S^1_{X_t}.$$

By the measurability lifting theorem (see Appendix) the correspondence $t \mapsto \{\tilde{y}(t) \in L_1(\mu, Y) : \tilde{y}(t, \omega) \in F(t, \omega, \tilde{x}) \;\; \mu - a.e.\}$ has a measurable graph and so does $t \mapsto S^1_{X_t}$ by (A.4). Thus, for each fixed $\tilde{x} \in S^1_X, \varphi(\cdot, \tilde{x})$ has a measurable graph. Since for each fixed $\tilde{x} \in S^1_X, F(\cdot, \cdot, \tilde{x})$ has a measurable graph and it is nonempty-valued then by the Aumann measurable selection theorem, it admits a measurable selection and we can conclude that φ is nonempty valued. It follows from the convex valueness of F that φ is also convex valued. By Diestel's theorem (see Appendix) the set $S^1_{X_t}$ is a weakly compact subset of $L_1(\mu, Y)$ and so is S^1_X. Notice that S^1_X is a metrizable set as being a weakly compact subset of the separable Banach space $L_1(\nu, L_1(\mu, Y))$, (Dunford-Schwartz [1958, p.434]). It follows from the u.s.c. lifting theorem (see Appendix) that for each fixed $t, \varphi(t, \cdot)$ is weakly u.s.c. Define the correspondence $\Phi : S^1_X \to 2^{S^1_X}$ by

$$\Phi(\tilde{x}) = \{\tilde{y} \in S^1_X : \tilde{y}(t) \in \varphi(t, \tilde{x}) \;\; \nu - a.e.\}.$$

Another application of the u.s.c. lifting theorem (see Appendix) enables us to conclude that Φ is a weakly u.s.c. correspondence which is obviously convex valued (since φ is convex valued) and also nonempty valued (recall once more the Aumann measurable selection theorem and notice that the set S^1_X is metrizable). Since the set S^1_X is weakly compact, convex and nonempty the Fan-Glicksberg theorem is applicable and there is a fixed point, i.e., there exist $\tilde{x}^* \in S^1_X$ such that $\tilde{x}^* \in \Phi(\tilde{x}^*)$, which implies that for $\nu - a.e.$, (i) $\tilde{x}^*(t, \omega) \in A(t, \omega, \tilde{x}^*) \;\; \mu - a.e.$ and (ii) $U(t, \omega, \tilde{x}^*, \tilde{x}^*(t, \omega)) = \max_{y \in A(t,\omega,\tilde{x}^*)} U(t, \omega, \tilde{x}^*, y) \;\; \mu - a.e.$ This completes the proof of the theorem.

4.2 Convexifying Effect

We will now show that the concavity assumption on the utility function can be dropped. The key condition that it is introduced is the assumption of "many more agents than strategies", that is, the dimensionality of the set of agents is greater than the dimensionality of the strategy space. In this set up, the set of joint strategies S^1_X, will now be replaced by the integral of the set-valued strategy correspondence X. As previously, let $S^1_{X_t} = \{y(t) \in L_1(\mu, Y) : y(t, \cdot) : \Omega \to Y \text{ is } \mathcal{F}_t - \text{measurable and } \ y(t,\omega) \in X(t,\omega) \quad \mu - a.e.\}$, and let $S^1_X = \{\tilde{y} \in L_1(\nu, L_1(\mu, Y)) : \tilde{y}(t) \in S^1_{X_t} \quad \nu - a.e.\}$. Define the integral of X as: $\int_T X \equiv \int_T X(t) d\nu(t) = \{\int_T \tilde{x} : \tilde{x} \in S^1_X\}$.

The *interim expected utility* of agent t, $\bar{U}(t, \omega, \cdot, \cdot) : \int_T X \times X(t,\omega) \to \mathcal{R}$ is defined as previously, but now the set of joint strategies has been replaced by the integral, $\int_T X$. The bar indicates that this interim expected utility has different domain than the one in Section 4.1. The idea of the proof is similar. However, we have to replace in the augment, the u.s.c. lifting theorems, with the fact that integration preserves u.s.c., and also make use of the the Rustichini-Yannelis (1991) theorem which proves that the integral of a Banach-valued correspondence is convex provided that the dimensionality of the space of agents is bigger than the dimensionality of the strategy space. All these technical details can be found in the Appendix.

Definition 4.2: A *(convexifying) social asymmetric equilibrium* for Γ is a strategy profile $\tilde{x}^*$ in $\int_t X$, i.e., there exist $\tilde{x}(t) \in S^1_{X_t} \quad \nu - a.e.$, $\int_T \tilde{x} = \tilde{x}^*$, such that, for $\nu - a.e.$,

(i) $\tilde{x}(t,\omega) \in A(t,\omega,\tilde{x}^*) \quad \mu - a.e.$, and

(ii) $\bar{U}(t,\omega,\tilde{x}^*,\tilde{x}(t,\omega)) = \max_{y\in A(t,\omega,\tilde{x}^*)} \bar{U}(t,\omega,\tilde{x}^*,y) \quad \mu - a.e.$

The idea of a convexifying equilibrium is not new, for a different framework it was introduced by Haller [1993].[3] Haller works with a finite dimensional strategy space, thus, he does have "many more agents than strategies" implicitly in his model. Haller [1993], also considers infinite dimensional strategy spaces but the concaving assumption cannot be dispenced with, because in his framework the Lyapunov theorem fails (i.e., the dimension assumption (A.5) below, may not hold).

The notion of the equilibrium is the same as in definition 4.1, except that now condition (ii) indicates that no agent can deviate from his/her optimal strategy and increase his/her payoff. However, all other player's strategy is captured by the integral (average sum of the optimal individual strategies), i.e., $\tilde{x}^* = \int_T \tilde{x}$ and the effect of each individual's strategy $\tilde{x}(t,\cdot)$ on the aggregate (average) is simply zero. This is in essence the idea of perfect competition, which is captured by our model. Notice that in definition 4.1, we are dealing with products and there is no convexifying effect on aggregation.

The assumption of "many more agents than strategies" is formally stated:

(A.5) The pair $((T,\tau,\nu),Y)$ satisfies the condition of Theorem A in the Appendix, i.e., if $E \in \tau, \nu(E) > 0$, then $\dim L_{\infty,E}(\nu) > \dim Y$.

[3] An early version was circulated as a working paper at VPI in 1986.

Theorem 2: Let Γ be a social system with asymmetric information satisfying (A.1), (A.2), (a), (b), (d) and (A.3) - (A.5). Then a convexifying social asymmetric equilibrium exists in Γ.

Proof: As in the proof of Theorem 1, the continuity and measurability properties of the interim expected utility $\bar{U} : T \times \Omega \times \int_T X \times X(t,\omega) \to \mathcal{R}$ are still valid, but now for each $(t,\omega,\tilde{x}) \in T \times \Omega \times \int_T X, \bar{U}(t,\omega,\tilde{x},\cdot)$ is not concave. We will show below that this is not a problem.

Define $\bar{A} : T \times \Omega \times \int_T X \to 2^Y$ by $\bar{A}(t,\omega,\tilde{x}) = A(t,\omega,x)$.

Define $F : T \times \Omega \times \int_T X \to 2^Y$ by

$$F(t,\omega,\tilde{x}) = \{y \in \bar{A}(t,\omega,\tilde{x}) : \bar{U}(t,\omega,\tilde{x},y) = \max_{z \in \bar{A}(t,\omega,\tilde{x})} \bar{U}(t,\omega,\tilde{x},z)\}.$$

As in the Proof of Theorem 1, F is nonempty valued, recall that $\bar{U}(t,\omega,\tilde{x},\cdot)$ is weakly u.s.c. and $\bar{A}$ is weakly compact valued so the maximum is obtained. As in the previous theorem, for each fixed $(x,\omega) \in T \times \Omega, F(t,\omega,\cdot) : \int_T X \to 2^Y$ is weakly u.s.c. and for each fixed $\tilde{x} \in \int_T X, F(\cdot,\cdot,\tilde{x}) : T \times \Omega \to 2^Y$ has a measurable graph.

Define the correspondence $\varphi : T \times \int_T X \to 2^{L_1(\mu,Y)}$ by

$$\varphi(t,\tilde{x}) = \{\tilde{y}(t) \in L_1(\mu,Y) : \tilde{y}(t,\omega) \in F(t,\omega,\tilde{x}) \quad \mu - a.e.\}.$$

For each fixed $t \in T, \varphi(t,\cdot)$ is weakly u.s.c. and for each fixed $\tilde{x} \in \int_T X, \varphi(\cdot,\tilde{x})$ has a measurable graph (see Appendix). Define $\Phi : \int_T X \to 2^{\int_T X}$ by $\Phi(\tilde{x}) = \int_T \varphi(t,\tilde{x})$. Then ϕ is weakly u.s.c., nonempty valued (recall Aumann's measurable selection theorem), and by Theorem A in the Appendix it is convex. As noted in the Appendix, $\int_T X$ is weakly compact and clearly convex, metrizable (recall that weakly compact subsets of a separable Banach space are metrizable), and it is also, nonempty valued (recall assumption (A.4) and the Aumann measurable selection theorem). Thus, by the Fan-Glicksberg fixed point theorem there exist $\tilde{x}^* \in \int_T X$ such that $\tilde{x}^* \in \Phi(\tilde{x}^*)$. It can be easily checked that $\tilde{x}^*$ is a convexifying social asymmetric equilibrium for Γ.

5 Ex ante Expected Utility

Notice that both theorems in Sections 4.1 and 4.2 are obtained for interim expected utility functions, and this is the Bayesian framework where agents receive a signal as to what is the event in their partition which contains the realized state of nature. Thus, agents condition their expected utility on the event which contains the realized state of nature and update their priors using Bayes' rule. In this set up the (interim) conditional expected utility depends on the realized state of nature and the arguments required to prove Theorems 1 and 2 are a bit different than the ex ante case we consider below. In the ex ante case there is no signaling and agents integrate their utility function over any possible state of nature. Thus, the (ex ante) expected utility doesn't depend on the states of nature. In particular, *the ex ante expected utility* $\bar{\bar{U}} : T \times S^1_X \times S^1_{X_t} \to \mathcal{R}$ is defined as

$$\bar{\bar{U}}(t, \tilde{x}, x_t) = \int_\Omega u(t, \tilde{x}(\omega), x_t(\omega)) d\mu(\omega).$$

We can now define the corresponding notion of Definition 4.1 for the ex ante expected utility.

Definition 5.3: An *(ex ante) social equilibrium* for Γ is a strategy profile $\tilde{x} \in S^1_X$ such that for $\nu - a.e.$,

(i) $\tilde{x} \in S^1_A(t, \tilde{x}) = \{y \in S^1_{X_t} : y(t, \omega) \in A(t, \omega, \tilde{x}) \quad \mu - a.e.\}$, and

(ii) $\bar{\bar{U}}(t, \tilde{x}, \tilde{x}_t) = \max_{z \in S^1_A(t,\tilde{x})} \bar{\bar{U}}(t, \tilde{x}, z)$.

Theorem 3: Let Γ be a social system with asymmetric information satisfying (A.1) - (A.4). Then there exists an ex ante social equilibrium in Γ.

Proof: The idea of the proof remains essentially the same. Some modifications are required. We outline the argument.

Define $S^1_A : T \times S^1_X \to 2^{L_1(\mu, Y)}$ by

$$S^1_A(t, \tilde{x}) = \{y \in S^1_{X_t} : y(t, \omega) \in A(t, \omega, \tilde{x}) \quad \mu - a.e.\}.$$

By the measurability lifting theorem for each fixed $\tilde{x} \in S^1_X, S^1_A(\cdot, \tilde{x})$ has a measurable graph and by Corollary 5.6 in Yannelis (1991, p. 21), for each fixed $t \in T, S^1_A(t, \cdot)$ is (weakly) continuous. It follows from the Aumann measurable selection theorem that S^1_A is nonempty valued (recall that A has a measurable graph).

Define the correspondence $F : T \times S^1_X \to 2^{L_1(\mu, Y)}$ by

$$F(t, \tilde{x}) = \{y \in S^1_A(t, \tilde{x}) : \bar{\bar{U}}(t, \tilde{x}, y) = \max_{z \in S^1_A(t,\tilde{x})} \bar{\bar{U}}(t, \tilde{x}, z)\}.$$

As before, for each fixed $t \in T, F(t, \cdot)$ is weakly u.s.c. and for each fixed $\tilde{x} \in S^1_X, F(\cdot, \tilde{x})$ has a measurable graph. Furthermore, F is nonempty and convex valued. Define the correspondence $\Phi : S^1_X \to 2^{S^1_X}$ by $\Phi(\tilde{x}) = \{\tilde{y} \in S^1_X : \tilde{y}(t) \in F(t, \tilde{x}) \quad \nu - a.e.\}$. By the u.s.c. lifting theorem (Appendix) Φ is weakly u.s.c. with convex, nonempty values (recall Aumann's measurable selection theorem) and obviously, Φ maps points from a weakly compact (Diestel's theorem) convex, nonempty set S_X^1 into sets of subsets of S_X^1. By the Fan-Glicksberg fixed point theorem there exist $\tilde{x}^* \in S_X^1$ such that $\tilde{x}^* \in \Phi(\tilde{x}^*)$. It can be easily checked that $\tilde{x}^*$ is an ex ante social equilibrium for Γ.

5.1 Convexifying Effect (Ex ante Expected Utility)

We will now show that by imposing condition (A.5), i.e., many more agents than strategies we can dispence with the concavity assumption on the utility functions. Thus, the counterpart of Theorem 2 for the ex ante case can be obtained.

We first need the following definition:

Definition 5.1: An *ex ante convexifying social equilibrium* for Γ is a strategy profile $\tilde{x}^* \in \int_T X$, i.e., there exist $\tilde{x}(t) \in S^1_{X_t}, \nu - a.e., \int_T \tilde{x} = \tilde{x}^*$, such that, for $\nu - a.e.$ conditions (i) and (ii) of definition 5.3. hold.

Theorem 4: Let Γ be a social system with asymmetric information satisfying (A.1), (A.2), (a), (b), (d), (A.3) - (A.5). Then a convexifying ex ante social asymmetric equilibrium exists in Γ.

Proof: We outline the basic argument. Define the correspondence $S^1_A : T \times \int_T X \to 2^{L_1(\mu,Y)}$ by

$$S^1_A(t, \tilde{x}) = \{y \in S^1_{X_t} : y(t, \omega) \in A(t, \omega, x) \quad \mu - a.e.\}.$$

Then for each fixed $\tilde{x} \in \int_T X, S^1_A(\cdot, \tilde{x})$ has a measurable graph and for each fixed $t \in T, S^1_A(t, \cdot)$

is weakly continuous, convex. Furthermore, $S^1_A(\cdot,\cdot)$ nonempty valued. Define the set-valued function $F : T \times \int_T \to 2^{L_1(\mu,Y)}$ by

$$F(t,\tilde{x}) = \{y \in S^1_A(t,\tilde{x}) : \bar{\bar{U}}(t,\tilde{x},y) = \max_{z\in S^1_A(t,\tilde{x})} \bar{\bar{U}}(t,\tilde{x},z)\}.$$

For each fixed $t \in T$, $F(t,\cdot)$ is weakly u.s.c., nonempty valued and for each fixed $\tilde{x} \in \int_T X$, $F(\cdot,\tilde{x})$ has a measurable graph. Define the correspondence $\Phi : \int_T X \to 2^{\int_T X}$ by $\Phi(\tilde{x}) = \int_T F(t,\tilde{x})$.

As noted earlier in the proof of Theorem 2, $\int_T X$ is weakly compact, convex, and nonempty. Furthermore, the correspondence Φ is weakly u.s.c. nonempty valued and by Theorem A in the Appendix, convex valued. Thus, there is a fixed point which can be easily checked that it is an ex ante convexifying social equilibrium for Γ.

6 Non-Convex Strategy Sets (Pure Strategies)

We will now show how condition (A.5) plays an important role to even dispence with the assumption of convex valued strategy correspondences, i.e. assumption (A.1) (a) can be replaced by

(A.1)

(a′) $X : T \times \Omega \to 2^Y$ is a nonempty, weakly compact valued and integrably bounded correspondence.

Denote by $X^e(\cdot,\cdot)$ the *set of all the extreme points of* $X(\cdot,\cdot)$.

Define the sets $S^1_{X^e_t} = \{y(t) \in L_1(\mu, Y) : y(t,\cdot) : \Omega \to 2^Y$ is $\mathcal{F}_t$-measurable and $y(t,\omega) \in X^e(t,\omega) \quad \mu - a.e.\}$, and $S^1_{X^e} = \{\tilde{y} \in L_1(\nu, L_1(\mu, Y)) : \tilde{y}(t) \in S^1_{X^e_t} \quad \nu - a.e.\}$.

Define $\int_T X^e = \{\int_T \tilde{z} : \tilde{z} \in S^1_{X^e}\}$.

Definition 6.1: A *pure strategy social asymmetric equilibrium* for Γ is a strategy profile $\tilde{x}^*$ in $\int_T X^e$, i.e., there exist $x(t) \in S^1_{X^e_t} \quad \nu - a.e.$, $\int_T \tilde{x} = \tilde{x}^*$, such that, for $\nu - a.e.$, conditions (i) and (ii) of Definition 4.2. hold.

The following Lemma is the key result needed for the proof of the existence of a pure strategy social asymmetric equilibrium.

Lemma 6.1 (*Purification*): Let Γ be a social system with asymmetric information satisfying (A.1), (a′), (b) and (A.5). Then $\int_T \overline{\text{con}} X^e = \int_T X^e = \int X$ and this set is weakly compact, convex and nonempty.

Proof: It follows from the Krein-Milman theorem (see for example Aliprantis-Burkinshaw (1985) or Dunford-Schwarz (1958)) that

$$\overline{\text{con}} X^e(t,\omega) = X(t,\omega) \text{ for } \nu - a.e. \text{ and } \mu - a.e. \qquad (*)$$

Notice that in view of $(*)$ above, the set $S^1_{\overline{\text{con}} X^e_t} = \{y(t) \in L_1(\mu, Y) : y(t,\cdot) : \Omega \to 2^Y$ is $\mathcal{F}_t$-measurable and $y(t,\omega) \in \overline{\text{con}} X^e(t,\omega) \quad \mu - a.e.\}$, is equal to $S^1_{X_t}$.

Since $S^1_X = \{\tilde{y} \in L_1(\nu, L_1(\mu, Y)) : \tilde{y}(t) \in S^1_{X_t} \quad \nu - a, .e.\} = \{\tilde{y} \in L_1(\nu, L_1(\mu, Y)) : \tilde{y}(t) \in S^1_{\overline{\text{con}} X^e_t} \quad \nu - a.e.\} = S^1_{\overline{\text{con}} X^e}$, it follows that $\int_T X = \{\int_T \tilde{x} : \tilde{x} \in S^1_X\} = \int_T \overline{\text{con}} X^e = \{\int_T \tilde{x} : \tilde{x} \in S^1_{\overline{\text{con}} X^e}\}$.

By Theorem A in the Appendix, we have that

$$\int_T \overline{\text{con}} X^e = \int_T X^e,$$

and therefore, $\int_T X^e = \int_T X$. The weak compactness and nonemptyness follow the same way as in the proof of Theorem 2.

In view of the above Lemma one can easily establish the existence of a pure strategy social asymmetric equilibrium for Γ, by repeating the same argument used to prove Theorem 2. Notice that no convexity assumption is needed on the strategy sets. Formally we have the following result.

Theorem 5: Let Γ be a social system with asymmetric information satisfying (A.1), (a′), (b), (A.2), (a), (b), (d) and (A.3) - (A.5). Then a pure strategy social asymmetric equilibrium exists in Γ.

7 Concluding Remarks and Open Questions

7.1: Throughout the paper we assumed that the state space Ω is countable. The reason for this was to prove the (weak) continuity of the expected utility. It turns out that if Ω is uncountable, then the expected utility is (weakly) continuous if and only if the utility function is affine (see Balder-Yannelis (1993)). It seems that in an expected utility set up, (either interim or ex ante) the affine linearity of the utility function is a rather strong assumption and it is less acceptable than the assumption of a countable state space.

7.2: The strategy space in this paper was assumed to be a separable Banach space. Separability seems to play an important role in the proofs as the measurable selection theorem is applied extensively. It is not clear how one can go beyond the separable case. This seems to be an open question.

7.3: Notice that the space of measure on a compact metric, endowed with the weak* topology is separable metric. Hence, if one employs weak* measurable selection theorems as well as the Gelfand integration, then the counterparts of all the theorems in this paper can be established for the space of measures. This is of interest in order to examine models of monopolistic competition or differentiated commodities.

7.4: It is well known (see for example, Arrow-Debreu (1954)), that the deterministic social equilibrium existence theorem can be used to prove the existence of a Walrasian equilibrium. One can follow the same idea to use the results of this paper to prove expected or Bayesian Walrasian equilibrium theorems for economies with asymmetric information and with a continuum of agents. It is our conjecture that, by using the theorems in this paper, the recent results of Cornet-Topozu (forthcoming) and Balder (2004) can be extended to cover asymmetric information. Moreover, the Bayesian Walrasian equilibrium existence theorems in Balder-Yannelis (2005), Podczeck-Yannelis (2005) and Podczeck-Tourky -Yannelis (2005) can be extended to a continuum of agents. As the moment this seems to be an open question.

8 Appendix

The results below have been used in the proof of our main theorem. We refer the reader to Yannelis [1991] for more details and further references.

Aumann Measurable Selection Theorem. *Let* (T, τ, μ) *be a complete finite measure space,* Y *be a complete, separable metric space and* $\phi : T \to 2^Y$ *be a nonempty valued correspondence with a measurable graph, i.e.,* $G_\phi \in \tau \otimes \beta(Y)$. *Then there is a measurable function* $f : T \to Y$ *such that* $f(t) \in \phi(t)$ $\mu - a.e.$

Diestel's Theorem. *Let* (T, τ, μ) *be a complete finite measure space,* X *be a separable Banach space and* $\phi : T \to 2^X$ *be an integrally bounded, convex, weakly compact and nonempty valued correspondence. Then* S^1_ϕ *is weakly compact in* $L_1(\mu, X)$.

Proof: See Yannelis [1991].

U.S.C. Lifting Theorem. Let Y be a separable space, $(\Omega, \mathcal{F}, \mu)$ be a complete finite measure space and $X : \Omega \to 2^Y$ be an integrably bounded, nonempty, convex valued correspondence such that for all $\omega \in \Omega, X(\omega)$ is a weakly compact, convex subset of Y. Denote by S^1_X the set $\{x \in L_1(\mu, Y) : x(\omega) \in X(\omega) \ \ \mu - a.e.\}$. Let $\phi : \Omega \times S^1_X \to 2^Y$ be a nonempty, closed, convex valued correspondence such that $\phi(\omega, x) \subset X(\omega)$ for all $(\omega, x) \in \Omega \times S^1_X$. Assume that for each fixed $x \in S^1_X, \phi(\cdot, x)$ has a measurable graph and that for each fixed $\omega \in \Omega,$ $\phi(\omega, \cdot) : S^1_X \to 2^Y$ is u.s.c. in the sense that the set $\{x \in S^1_X : \phi(\omega, x) \subset V\}$ is weakly open in S^1_X for every norm open subset V of Y. Define the correspondence $\Phi : S_X^1 \to 2^{S_X^1}$ by

$$\Phi(x) = \{y \in S_X^1 : y(\omega) \in \phi(\omega, x) \;\; \mu - a.e.\}.$$

Then Φ is weakly u.s.c., i.e., the set $\{x \in S_X^1 : \Phi(x) \subset V\}$ is weakly open in S_X^1 for every weakly open subset V of S_X^1.

Proof: See Yannelis [1991].

Measurability Lifting Theorem: Let Y and E be separable Banach spaces, and (T, τ, ν) and $(\Omega, \mathcal{F}, \mu)$ be finite complete separable measure spaces. Let $\gamma : T \times \Omega \times E \to 2^Y$ be a nonempty valued correspondence. Suppose that for each $y \in E$, $\gamma(\cdot, \cdot, y)$ has a measurable graph. Define the correspondence $\psi : \Omega \times E \to 2^{L_1(\mu, Y)}$ by

$$\psi(t, y) = \{x(t) \in L_1(\mu, Y) : x(t, \omega) \in \gamma(t, \omega, y) \;\; \mu - a.e.\}.$$

Then for each $y \in E, \psi(\cdot, y)$ has a measurable graph.

Proof: See Balder-Yannelis (1991).

Denote by $L_\infty(\nu)$ the space of real valued measurable, essentially bounded functions derived on (T, τ, ν). For any $E \in \tau$ the measure space (E, τ_E, ν_E) is naturally defined, and so is the space $L_{\infty,E}(\tau) = \{f : E \to \mathcal{R}, f$ is τ_E-measurable and ν_E-essentially bounded$\}$.

For any vector space over the real field an algebraic (Hamel) basis exists. The cardinality of any Hamel basis is the same, and denote for any vector space Y, $\dim Y$ the cardinality of any of its bases.

Theorem A (*Convexifying effect*): Let K be a weakly compact subset of a Banach space Y, and let $\varphi : T \to 2^K$ be a lower measurable closed valued correspondence. Suppose that for any pair $((T,\tau,\nu),Y)$ if $E \in \tau, \nu(E) > 0$, then $\dim L_{\infty,E}(\nu) > \dim Y$.

Then $\int \varphi = \int \overline{\text{con}}\varphi$, (where $\overline{\text{con}}$ denotes, closed convexhull).

Proof: See Rustichini-Yannelis (1991, Main theorem).

The theorem above is an infinite generalization of Theorem 3 in Aumann (1964). It should be noted that the assumption that φ is lower measurable and closed valued implies that φ has a measurable graph (see for example Castaing-Valadier (1977)).

Define the mapping $\psi : L_1(\nu, X) \to X$ by $\psi(x) = \int_T x(t)d\nu(t), \psi$ is linear and norm continuous and thus weakly continuous (Dunford-Schwartz (1958, p.422)). Denote by S^1_φ the set of all Bochner integrable selections from the correspondence $\varphi : T \to 2^X$, i.e.,

$$S^1_\varphi = \{y \in L(\nu, X) : y(t) \in \varphi(t) \quad \nu - a.e.\}.$$

The integral of φ is $\psi(S^1_\varphi) = \{\psi(x) : x \in S^1_\varphi\} = \int \varphi$. In view of the above, the reader can easily conclude that the counterparts of the u.s.c. and measurability lifting theorems for the set of all Bochner integrable selections of a correspondence, also hold for the integral of a correspondence. This is also the case for the weak compactness of the integral of a correspondence, i.e., by Diestel's theorem it follows that S^1_φ is weakly compact, and therefore, $\psi(S^1_\varphi) = \int \varphi$, i.e., the *integral of φ is also weakly compact.*

Integration preserves u.s.c. theorem: Let (T, τ, ν) be a complete finite measure space, P

be a metric space and Y be a separable Banach space. Let $\psi : T \times P \to 2^Y$ be a nonempty, compact valued correspondence such that for each fixed $t \in T, \psi(t, \cdot)$ is u.s.c. and that for each $p \in P, \psi(\cdot, p)$ has a measurable graph. Then $\int_T \psi(t, \cdot)$ is u.s.c.

Proof: See Yannelis (1991, Theorem 6.6 and Remark 6.1).

A version of the above theorem for weak-u.s.c. also holds, (see Yannelis (1991, Theorem 5.5)).

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