Policy Implementation under Endogenous Time Inconsistency[*]

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Abstract

The paper considers policy implementation in a two-party political system. We show that if it is more likely for the current ruling party to be in office in the next period than the opposition party, the government naturally possesses generalized hyperbolic discounting so that it is faced with the time inconsistency problem. The time inconsistent government appears to lack incentive to implement the policy to undertake the project with immediate costs and long-lasting benefits especially if the costs are large. We show, however, there always exists a subgame perfect equilibrium in which the project is undertaken as long as it is socially beneficial. If the implementation costs are very large, the project must be carried out gradually and the process must continue indefinitely.

Preliminary version.

1 Introduction

Individuals often procrastinate doing things that generate lasting benefits with an immediate cost, to the detriment of their long-term interests. Quitting smoking, alcohol, and other addictions is just an example. A recent literature (e.g., Akerlof, 1991 and O'Donoghue and Rabin, 1999) explains this phenomenon by focusing on the existence of present-biased preferences, which induce time-inconsistent behavior. A present-biased, time-inconsistent individual may procrastinate completing a task forever, even though it is in her best long-term interest to complete the task immediately.

Similarly, it is often observed that politicians avoid implementing policies that generate long-lasting benefits with immediate costs. Raising income taxes is extremely unpopular for politicians even if it benefits citizens in the long-run by reducing the government deficit and hence lowering the long-term interest rate. Tariff reduction is also unpopular despite of its long-term benefits to the country as a whole, not least because relocation costs resulting from an induced sectoral adjustment from import good sectors to export good sectors are incurred immediately while social benefits are spread far into the future. Politicians, who care more about the present than the future, naturally put more weights on the welfare of contemporary constituents than those of future constituents. Therefore, they may well resist raising income taxes, trade liberalization, etc.

Of course, it is not surprising that if the implementation costs are large, the net benefit of the policy may be negative and hence the policy would not be and should not be implemented. We show, however, that even if the net benefit is positive for all individuals, the government may procrastinate about implementing the policy in a two-party political system, such as in the United States or Britain, in which two parties alternate in taking office. Each party puts more weight on the social benefits derived from the project when it is in office, while it discounts the social benefits when it is out of office. We show that in such a two-party political system, the government of any period will be faced with a present-biased, generalized hyperbolic utility function, so that its behavior is constrained by time inconsistency.[1]

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[1] Amador (2003) demonstrates that in a similar two-party political environment, the party in office will

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We demonstrate that the present-biased governments may (i) carry out the project immediately exactly in accordance with voters' interests, (ii) procrastinate somewhat, but still manage to complete the whole project in some period within a finite time, (iii) undertake the project in stages and the process continues indefinitely, or (iv) completely fail to undertake the socially beneficial project. Which outcome arises in equilibrium depends on the cost of the project relative to the discount factor. We emphasize that the outcome of policy implementation is broadly applicable to many situations in which present-biased individuals complete a task that generate long-lasting benefits with immediate costs.

2 The Basic Setup of the Model

There are two political parties that seek power in the government. One of them is in office in period $t \in \{0, 1, 2, \cdots\}$. The party in office makes political decisions in accordance with its own preferences, so the objective function of the current government is the same as that of the party in office. The two parties have the same preferences over the policy that we consider and the same discount factor δ, which is also the same as that of voters. The selection of the ruling party in each election is characterized by a Markov process, such that the current ruling party will also be in office in the next period with probability $p > 1/2$. That is, we assume that the ruling party has a higher probability to be in office in the next period than other parties. We argue in the Concluding remarks that voters have incentives to re-elect the incumbent to mitigate the government's time-inconsistency problem. We would obtain similar results even if we rule out the advantage of being the incumbent in the next election.

The policy that we consider is to undertake a project that involves immediate costs of c but generates a constant benefit of 1 in every period thereafter. The project can be carried out gradually so that the fraction a_t of the project undertaken in period t immediately imposes the costs $a_t c$ to society while generating a flow benefit of a_t. We assume that

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have a quasi-hyperbolic utility function. Our argument can easily be generalized to the case of multi-party political system with more than two parties. We demonstrate our argument in the case of two parties to avoid the discussion of the problem about coalition formation to gain a majority, etc., which are not of central interest of our analysis.

$1/(1 - \delta) > c$, so the project is worth carrying out from the voters' viewpoint.

The net benefit to society in period t is given by

$$u_t = \sum_{k=0}^{t} a_k - a_t c. \tag{1}$$

The first term on the right-hand side shows the benefit that society enjoys in period t from the fraction of the project that have been completed, whereas the second term represents the costs that society incurs from part of the project undertaken in period t. We assume that the party in office puts a (normalized) weight of one on social welfare, and so its per-period payoff equals u_t, while the opposition party puts a weight of $\alpha \in [0, 1]$ on social welfare. This discounting is motivated by the presumption that relative to the party in office, the opposition party is indifferent to voters' well-being perhaps due to lack of responsibility. We shall show that the ruling party's objective function exhibits generalized hyperbolic discounting and therefore policy implementation is constrained by time-inconsistency.

3 Endogenous Time Inconsistency

In this section, we show that in two-party politics, the party in office will possess a payoff function with generalized hyperbolic discounting. By generalized hyperbolic discounting, we mean one such that the discounting between two consecutive periods t and $t + 1$ diminishes as t increases. To be more specific, let

$$U_t = \sum_{k=0}^{\infty} \beta_k u_{t+k} \tag{2}$$

represent the intertemporal payoff function for the party in office in period t, which we call Government t henceforth. Then, U_t exhibits generalized hyperbolic discounting if the ratio of the two consecutive discount functions β_{k+1}/β_k increases with k.[2]

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[2] The instantaneous discount rate of the "usual" exponential discount function $\beta_e(t) \equiv e^{-rt}$ in continuous time models is given by $-\beta'_e(t)/\beta_e(t) = r$, whereas that of the hyperbolic discount function $\beta_h(t) \equiv (1 + \alpha t)^{-\gamma/\alpha}$ is given by $-\beta'_h(t)/\beta_h(t) = \gamma/(1 + \alpha t)$ that decreases with t (for hyperbolic discounting, see Loewenstein and Prelec, 1992, who call it *generalized* hyperbolic discounting contrary to our terminology). Phelps and Pollak (1968) develop an intertemporal utility function of the form: $U_t = u_t + \beta \sum_{k=1}^{\infty} \delta^k u_{t+k}$ (where $0 < \beta < 1$ and $0 < \delta < 1$) to capture imperfect altruism for future generations. Laibson (1997) introduces this utility function with quasi-hyperbolic discounting to the behavioral economics in order to

Let p_k denote the probability that the current ruling party will also be in office k periods later. Since the party in office will be in office in the next period with probability p and the other party will be in office with probability $1 - p$, p_k evolves as

$$p_{k+1} = p \cdot p_k + (1 - p)(1 - p_k)$$
$$= 1 - p + (2p - 1)p_k, \tag{3}$$

with $p_0 = 1$. Figure 1 depicts the transition of this probability. We see from the figure that as k increases, p_k decreases while the ratio of p_{k+1} to p_k,

$$\frac{p_{k+1}}{p_k} = \frac{1 - p}{p_k} + 2p - 1, \tag{4}$$

increases.

To find the appropriate discount function and show that it exhibits generalized hyperbolic discounting, we first express the expected per-period payoff in period $t + k$ for Government t as

$$p_k u_{t+k} + (1 - p_k)\alpha u_{t+k} = [\alpha + (1 - \alpha)p_k]u_{t+k}.$$

As (1) shows, u_{t+k} depends on the action of Government $t + k$ as well as those of all governments before $t + k$. From the perspective of the party in office in period t, Government $t + k$ (for $k \geq 1$) may or may not be itself, so u_{t+k} depends on its own future action with probability p_k while u_{t+k} depends on the other party's future action with probability $1 - p_k$. Since both parties have the same preferences, Government t has no reason to distinguish between the two when it expects the action taken by Government $t + k$. Therefore, the expected per-period payoff can be written as in the above, and the intertemporal payoff for Government t is given by (2) in which

$$\beta_k \equiv \delta^k[\alpha + (1 - \alpha)p_k] \tag{5}$$

is the discount function applied to the social welfare k periods from t. Note that $\beta_0 = 1$ as $p_0 = 1$.

capture important properties of hyperbolic discounting. Note that the quasi-hyperbolic discounting is an example of the generalized hyperbolic discounting as β_{k+1}/β_k weakly increases with k ($\beta_1/\beta_0 = \beta\delta$ and $\beta_{k+1}/\beta_k = \delta$ for $k \geq 1$).

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This payoff function exhibits generalized hyperbolic discounting if

$$\frac{\beta_{k+1}}{\beta_k} = \frac{\delta \left[\frac{\alpha}{p_k} + (1-\alpha)\frac{p_{k+1}}{p_k} \right]}{\frac{\alpha}{p_k} + 1 - \alpha} \tag{6}$$

increases with k. To show that it is indeed the case, we first notice that both α/p_k and p_{k+1}/p_k increase as k increases. It is easy to see that, for any given $p_{k+1}/p_k < 1$, β_{k+1}/β_k increases as α/p_k increases. Since p_{k+1}/p_k also increases with k, we conclude that β_{k+1}/β_k increases with k and hence the government's payoff function exhibits generalized hyperbolic discounting.

It is also easy to see that β_{k+1}/β_k converges to δ as t increases. As Figure 1 indicates, p_{k+1}/p_k converges to 1 as k increases. Then, it follows immediately from (6) that β_{k+1}/β_k converges to δ. Note also from (3) that p_t decreases to $1/2$, which shows an important observation that the current ruling party keeps losing the advantage of being the incumbent and it loses the advantage (almost) completely far off in the future.

To gain a better understanding of why the two-party politics yields generalized hyperbolic discounting, we temporarily consider an alternative setting adopted by Amador (2003) in which party j will be in office with a fixed probability p^j in every period regardless of whether or not party j was in office in the last period. Then the discount function that is a counterpart of (5) is given by $\beta_k' \equiv \delta^k[\alpha + (1-\alpha)p^j]$, and therefore its ratio between consecutive periods β_{k+1}/β_k is $\delta[\alpha + (1-\alpha)p^j]$ for $k = 0$ and δ for any $k \geq 1$. This alternative political system generates the quasi-hyperbolic discounting (Laibson, 1997; see also footnote 2). The current ruling party discounts the social welfare in the next period more heavily than δ as it will be out of office with probability $1 - p^j$. Since the probability of being in office stays the same from the next period, i.e., the party in office never enjoys the advantage of being the incumbent in future elections, discounting between future consecutive periods is stationary.

4   Policy Implementation

In this section, we analyze the optimal policy choice of a government. Careful readers easily see that the same analysis can be applied to the problem of completing a task for an

individual whose utility function exhibits generalized hyperbolic discounting. It has been shown that an individual with a quasi-hyperbolic payoff function exhibits time-inconsistent behavior, which includes inefficient procrastination of costly actions that generate a future flow of large benefits. Now that the government, or the party in office, has a generalized hyperbolic payoff function, it is faced with a time inconsistency problem so that it may want to procrastinate. In this section, however, we show that (i) the entire project is carried out immediately in period 0 if the costs of the project are small, (ii) there may exhibits delay in undertaking the project if the costs are in the intermediate range, and (iii) the project is carried out gradually over indefinite periods of time if the costs are large although there also exists another equilibrium in which the project is never carried out.

Given the history $\{a_k\}_{k=0}^{t-1}$, Government t with the payoff function given in (2) chooses a_t under the constraint $\sum_{k=0}^{t} a_k \leq 1$. The action of Government t unambiguously affects those of future governments, and Government t's expectation about the actions of future governments affects its behavior. This policy implementation problem can be considered as a game played by the governments each of which lasts only one period.

Now, we rewrite Government 0's intertemporal payoff function given in (2) for $t = 0$, using the observation that the fraction a_t of the project undertaken in period t yields the expected net benefit $a_t \left(\sum_{k=0}^{\infty} \beta_{t+k} - \beta_t c \right)$:

$$U_0 = \sum_{t=0}^{\infty} \left[a_t \left(\sum_{k=0}^{\infty} \beta_{t+k} - \beta_t c \right) \right]. \tag{7}$$

Since (7) is linear with respect to $\{a_t\}_{t=0}^{\infty}$, it is the best for Government 0 that the project is carried out in the periods where the present value of the net benefit is greatest. That is, the best sequence of $\{a_t\}_{t=0}^{\infty}$ is that $a_t = 1$ if $t \in T^*$ and $a_t = 0$ if $t \notin T^*$, where

$$T^* = \arg \max_{t \in \{0,1,2,\cdots\}} \sum_{k=0}^{\infty} \beta_{t+k} - \beta_t c.$$

Generically, T^* is a singleton, so we write the generically unique element of T^* as t^*.

To find t^*, we compare the present values of net benefit for the two consecutive periods t and $t + 1$ and find

$$\sum_{k=0}^{\infty} \beta_{t+k} - \beta_t c > \sum_{k=0}^{\infty} \beta_{t+1+k} - \beta_{t+1} c \tag{8}$$

if and only if

$$\frac{\beta_{t+1}}{\beta_t} > \frac{c-1}{c}. \tag{9}$$

If neither party discounts the social welfare when it is out of office, i.e., $\alpha = 1$, then $\beta_t = \delta^t$ for any t, and the inequality (9) holds since it reduces to $1/(1-\delta) > c$. In this case, $\sum_{k=0}^{\infty} \beta_{t+k} - \beta_t c > \sum_{k=0}^{\infty} \beta_{t+1+k} - \beta_{t+1} c$ for all $t \geq 0$, so Government 0 prefers having the project undertaken in period t to having it postponed to the next period, no matter what t is. This implies $t^* = 0$, and it is in Government 0's best interest to carry out the entire project within its term. Note that, since $\beta_t = \delta^t$, the government's payoff function is exactly the same as that of the voters. Therefore, in this case, the government's action maximizes the welfare of the voters.

Proposition 1 *Suppose that neither party discounts the social welfare when it is out of office, i.e., $\alpha = 1$. Then the government in period 0 immediately completes the project, which accords with the voters' interest.*

On the other hand, if every party discounts the social welfare when it is out of office, i.e., $\alpha < 1$, postponing the project may be preferable for the current government. To see this point, we first observe

$$\frac{\beta_{t+k}}{\beta_t} = \Pi_{i=0}^{k-1} \frac{\beta_{t+i+1}}{\beta_{t+i}} > \Pi_{i=0}^{k-1} \frac{\beta_{i+1}}{\beta_i} = \frac{\beta_k}{\beta_0} = \beta_k,$$

where the inequality results from the generalized hyperbolic discounting. Thus, we have

$$\begin{aligned} \sum_{k=0}^{\infty} \beta_{t+k} - \beta_t c &= \beta_t \left(\sum_{k=0}^{\infty} \frac{\beta_{t+k}}{\beta_t} - c \right) \\ &> \beta_t \left(\sum_{k=0}^{\infty} \beta_k - c \right), \end{aligned}$$

which demonstrates that even if every government, including Government 0, has no incentive to carry out the project within its term, i.e., $\sum_{k=0}^{\infty} \beta_k - c < 0$, Government 0 may wish that the project be undertaken in the future period t, i.e., $\sum_{k=0}^{\infty} \beta_{t+k} - \beta_t c > 0$. The problem of policy implementation is much more subtle if $\alpha < 1$ than in the case of $\alpha = 1$ as we see shortly. The optimal timing of the implementation will depend on the costs of the project.

4.1 Low Implementation Costs

First, we consider the case in which the costs of the project are small such that $(c-1)/c \leq \beta_1 \equiv \delta\left[\alpha + (1-\alpha)p\right]$. In this case, it is worthwhile to undertake the project for any government as $\sum_{k=0}^{\infty} \beta_k - c > 0$. To see this claim, we first observe that for $k \geq 2$, $\beta_k = \beta_1 \Pi_{i=1}^{k-1} \frac{\beta_{i+1}}{\beta_i} > \beta_1^k$ due to the generalized hyperbolic discounting. Since $(c-1)/c \leq \beta_1$ is equivalent to $\sum_{k=0}^{\infty} \beta_1^k - c \geq 0$, the claim follows from the inequality $\beta_k > \beta_1^k$ (for $k \geq 2$). Now, as Figure 2 indicates, it is obvious that the inequality (9) holds for any t, so we have $t^* = 0$. The government of any period will undertake the entire remainder of the project if there is any. The unique subgame perfect equilibrium is that $a_0 = 1$ and $a_t = 1 - \sum_{k=0}^{t-1} a_k$ for any $t = 1, 2, \cdots$, so that Government 0 carries out the entire project despite of the generalized hyperbolic discounting.

Proposition 2 *If the costs of the project are small so that $(c-1)/c \leq \beta_1$, the entire project is carried out in period 0.*

4.2 Intermediate Implementation Costs

The government of any period prefers postponing the project if the costs of the project are in the intermediate range: $\beta_1 < (c-1)/c \leq \bar{\beta} \equiv \left(\sum_{k=0}^{\infty} \beta_k - 1\right)/\sum_{k=0}^{\infty} \beta_k$.

In this case, we have from $(c-1)/c \leq \bar{\beta}$ that $\sum_{k=0}^{\infty} \beta_k - c > 0$. Thus, the government of any period derives a positive net benefit from the part of the project that is undertaken by itself. Since $\beta_1 < (c-1)/c$, however, every government has an incentive to postpone the project. Figure 3 shows the situation in which $t^* = 2$. It is easy to see that the figure depicts the situation of $t^* = 2$, as the inequality (9) holds if and only if $t \geq 2$. In this example, the government of any period wishes that the project be undertaken two periods later. It appears that the project is at risk due to the time inconsistency problem.

However, there exists a subgame perfect equilibrium with cyclical strategies, in which the project is successfully undertaken. Cyclical strategy to complete a task with an immediate cost and infinite stream of delayed benefits has been introduced by O'Donoghue and Rabin

(2001) in the case of quasi-hyperbolic discounting.[3] In the following, we demonstrate that the strategy of the same type can implement the policy in this framework of generalized hyperbolic discounting.

Let us define $\hat{t}$ by

$$\hat{t} = \min\{t| \sum_{k=0}^{\infty} \beta_k - c > \sum_{k=0}^{\infty} \beta_{t+k} - \beta_t c\}.$$

Since $\sum_{k=0}^{\infty} \beta_{t+k} - \beta_t c$ increases with t until t^* is reached and then decreases with t to 0, we have $t^* < \hat{t} < \infty$. There are $\hat{t}$ subgame perfect equilibria such that for any $\tilde{t} \in \{0, 1, \cdots, \hat{t}-1\}$,

$$a_t = \begin{cases} 1 - \sum_{k=0}^{t-1} a_k & \text{if } t = \tilde{t} + i\hat{t}, \text{ for } i = 0, 1, 2, \cdots \\ 0 & \text{otherwise.} \end{cases} \tag{10}$$

Proposition 3 *If the costs of the project are in the intermediate range ($\beta_1 < (c-1)/c \le \bar{\beta}$), there are $\hat{t}$ subgame perfect equilibria such that the entire project is carried out in one of the periods $\{0, 1, \cdots, \hat{t} - 1\}$.*

Proof: When $t = \tilde{t} + i\hat{t}$, given that the fraction $1 - \sum_{k=0}^{t-1} a_k$ of the project remains to be undertaken, the government of that period would obtain the payoff (inclusive of the benefit from earlier actions) $U_t = \sum_{k=0}^{\infty} \beta_k - \left(1 - \sum_{k=0}^{t-1} a_k\right) c$ if it conforms to the equilibrium strategy. If it deviates by carrying out the fraction $a_t \in [0, 1 - \sum_{k=0}^{t-1} a_k)$, on the other hand, it would obtain

$$\left(\sum_{k=0}^{t} a_k\right) \sum_{k=0}^{\infty} \beta_k - a_t c + \left(1 - \sum_{k=0}^{t} a_k\right) \left(\sum_{k=0}^{\infty} \beta_{\hat{t}+k} - \beta_{\hat{t}} c\right),$$

since the fraction $1 - \sum_{k=0}^{t} a_k$ of the project is left to be undertaken in $\hat{t}$ periods later. The former is greater than or equal to the latter if and only if

$$\sum_{k=0}^{\infty} \beta_k - c \ge \sum_{k=0}^{\infty} \beta_{\hat{t}+k} - \beta_{\hat{t}} c.$$

Since this inequality holds by the definition of $\hat{t}$, we find that Government t conforms to the equilibrium strategy when $t = \tilde{t} + k\hat{t}$.

Next, we show that Government t also conforms to the equilibrium strategy when $t \ne \tilde{t} + k\hat{t}$. Let $s \in \{1, \cdots, \hat{t} - 1\}$ denote the number of periods that remain until the remainder

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[3]Matsuyama (1990) proposes cyclical strategy of the same type in a trade liberalization game.

of the project is to be undertaken. Then, for any given $\sum_{k=0}^{t-1} a_k$, the payoff for Government t when it conforms to the equilibrium strategy equals

$$\left(\sum_{k=0}^{t-1} a_k\right) \sum_{k=0}^{\infty} \beta_k + \left(1 - \sum_{k=0}^{t-1} a_k\right) \left(\sum_{k=0}^{\infty} \beta_{s+k} - \beta_s c\right),$$

whereas the payoff when it deviates by conducting $a_t \in (0, 1 - \sum_{k=0}^{t-1} a_k]$ of the remaining project equals

$$\left(\sum_{k=0}^{t} a_k\right) \sum_{k=0}^{\infty} \beta_k - a_t c + \left(1 - \sum_{k=0}^{t} a_k\right) \left(\sum_{k=0}^{\infty} \beta_{s+k} - \beta_s c\right).$$

The former is greater than the latter if and only if

$$\sum_{k=0}^{\infty} \beta_{s+k} - \beta_s c \geq \sum_{k=0}^{\infty} \beta_k - c,$$

which is satisfied for $s < \hat{t}$.

<div align="right">**Q.E.D.**</div>

Since $t^* < \hat{t}$, we immediately obtain the following corollary.

Corollary 1 *If the costs of the project are in the intermediate range, there exists a subgame perfect equilibrium in which the entire project is carried out in period $t^* > 0$ which is the optimal timing of the policy implementation for the government in period 0.*

We have shown that despite of the time inconsistency problem, the project can be successfully carried out. The voters wish that the project be carried out immediately since they possess the "usual" exponential discounting. Although there is such an equilibrium, the "focal" equilibrium may be the one in which the project is undertaken in the future period that is most preferable for the government in period 0.

4.3   High Implementation Costs

We finally consider the case in which $\bar{\beta} < (c-1)/c < \delta$. In this case, we have $\sum_{k=0}^{\infty} \beta_k - c < 0$ so that the government of any period would incur a loss from the part of the project that is carried out within its term. Nevertheless, every government wishes that the project be

undertaken sometime in the future since $\sum_{k=0}^{\infty} \beta_{t+k} - \beta_t c$ is positive if t is large enough. To see this claim, we note that

$$\sum_{k=0}^{\infty} \beta_{t+k} - \beta_t c = \beta_t \left[\sum_{k=0}^{\infty} \frac{\beta_{t+k}}{\beta_t} - c \right].$$

As we have seen in Section 3, the generalized hyperbolic discounting behave very similarly to the exponential discounting far off in the future, i.e., β_{k+1}/β_k converges to δ. Thus, β_{t+k}/β_t converges to δ^k, and hence the expression in the square brackets on the right-hand side of the above equation converges to $\sum_{k=0}^{\infty} \delta^k - c$ as t increases. Since $\sum_{k=0}^{\infty} \delta^k - c > 0$ under the assumption $1/(1-\delta) > c$, and since β_t remains positive for any t, we obtain that $\sum_{k=0}^{\infty} \beta_{t+k} - \beta_t c > 0$ if t exceeds a certain level.

Does any government have incentives to undertake some part of the project in this situation? It turns out that whether or not a government carries out part of the project crucially depends on the choice of other governments.

It is obvious from the above argument that any government would not wish to complete the project since it would incur a net loss from the last part of the project undertaken by itself. Thus, if all future governments are supposed to refrain from carrying out the project, the current government should also stay out of the project. The strategy profile in which $a_t = 0$ for any t is the subgame perfect equilibrium.

Proposition 4 *If the costs of the project are large such that $(c-1)/c > \bar{\beta}$, there is a subgame perfect equilibrium in which the project will not be carried out to the detriment of the voters' interests.*

This proposition is certainly bad news for the voters. Is not there another subgame perfect equilibrium in which some governments carry out at least part of the project? The cyclical strategies that we have considered in the last subsection would not work here since the government that is supposed to carry out the entire project certainly prefers obtaining zero payoff by doing nothing to obtaining a negative payoff by conforming to the prescribed cyclical strategy. The equilibrium payoff for any government that carries out part of the project must enjoy a nonnegative payoff since it can always stay away from the project.

Indeed, if the project is to be implemented at all, it must be spread out over time to assure a nonnegative payoff for every government. Consider the stationary strategy such that $a_t = a(1-a)^t$ for some constant $a \in (0,1)$. According to this strategy, every government undertakes the fraction a of the remainder of the project, and this process continues indefinitely. Now, regardless of its own action, every government receives a flow payoff from the part of the project that previous governments have completed. So we ignore this flow payoff when we examine the decision of the government. The relevant payoff for Government t equals

$$\sum_{i=0}^{\infty} \left[a(1-a)^{t+i} \left(\sum_{k=0}^{\infty} \beta_{i+k} - \beta_i c \right) \right]. \tag{11}$$

Since $\sum_{k=0}^{\infty} \beta_{i+k} - \beta_i c > 0$ if i is large enough, there exists $\bar{a}$ such that for any $a \in (0, \bar{a})$, the payoff (11) is positive.

Can this gradual implementation be supported by a subgame perfect equilibrium? Consider the following strategy profile:

$$a_t = \begin{cases} a(1-a)^t & \text{if there has been no deviation from } a_i = a(1-a)^i \text{ for all } i \leq t-1 \\ 0 & \text{otherwise.} \end{cases} \tag{12}$$

Notice again that the project will never be completed according to this strategy profile. No matter how small the remainder is, Government t would always be better off undertaking only the fraction a of the remainder. Thus, the governments would keep the project going forever. Indeed, the strategy profile (12) is a subgame perfect equilibrium.

Proposition 5 *If the costs of the project are large, the project can be carried out only if it is spread out over time. Indeed, there is a subgame perfect equilibrium in which every government carries out a fraction of the remainder of the project so the implementation process lasts indefinitely.*

Proof: We show here that the strategy profile (12) is subgame perfect. If there has been no deviation, Government t is supposed to select $a_t = a(1-a)^t$, obtaining a positive payoff from a_t. If Government t selects some other level of a_t, on the other hand, the equilibrium path would switch to the "punitive equilibrium" described in Proposition 4, making the

present value of the future payoff zero. Since the payoff for Government t derived from a_t (not including the flow payoff from the part completed before t) is nonpositive, the intertemporal payoff from a_t would be nonpositive if the government does not select $a_t = a \left(1 - a \right)^t$. Therefore, Government t will choose $a_t = a \left(1 - a \right)^t$ if there has been no deviation before period t.

Q.E.D.

There are also non-stationary, subgame perfect equilibria in which the policy implementation process lasts indefinitely. Consider a strategy profile $\{a_t\}_{t=0}^{\infty}$ such that

$$\sum_{i=0}^{\infty} \left[a_{t+i} \left(\sum_{k=0}^{\infty} \beta_{i+k} - \beta_i c \right) \right] > 0,$$

for any period t in which $a_t > 0$. We adopt a trigger strategy similar to the one above, except that any deviation of Government t that is supposed to select $a_t = 0$ is ignored. Any government with a positive a_t has no incentive to deviate for the same reason as the above. Any government with $a_t = 0$ has no reason to conduct a positive part since it would incur a loss from a_t for $\sum_{k=0}^{\infty} \beta_k - c < 0$.

5 Concluding Remarks

We have shown that in a two-party political system, the government will have the generalized hyperbolic discounting, so its preferences exhibit time inconsistency even though the preferences of a representative voter are time-consistent. We consider the timing and staging of implementation of a project which should be implemented immediately and completely if the representative voter's welfare is to be maximized. Time consistency will not be a problem in policy implementation if the costs of the project are small, in the sense that the governments' actions will be completely in line with welfare maximization of the representative voter, i.e., it implements the project immediately and completely. If the costs are in the intermediate range, however, the government of any period wishes that the project be undertaken by a future government. Even in this case, however, there is a cyclical equilibrium such that the entire project is carried out in a finite time. The project can also be undertaken when the

costs are large. In this case, the project must be carried out gradually and must continue indefinitely.

We have assumed that in the next election the current ruling party has the advantage of being the incumbent, i.e., the probability that the current ruling party will be re-elected for the next term is greater than a half ($p > 1/2$). We argue here that this assumption is reasonable as voters have incentives to re-elect the current ruling party to mitigate the government's time inconsistency problem. To this end, we first observe from (3) that $p_k = [(2p - 1)^k + 1]/2$ and hence

$$
\begin{aligned}
\beta_k &= \delta^k[\alpha + (1 - \alpha)p_k] \\
&= \frac{1 + \alpha}{2}\delta^k + \frac{1 - \alpha}{2}\delta^k(2p - 1)^k.
\end{aligned}
$$

Then, we have

$$
\sum_{k=0}^{\infty} \beta_k = \frac{1 + \alpha}{2(1 - \delta)} + \frac{1 - \alpha}{2[1 - \delta(2p - 1)]},
$$

which is increasing in p. That is, the higher the probability to be re-elected, the higher the present discounted sum of the benefits from the project. Since it is more likely that the government undertakes the project if this present value of the benefits is large, the time inconsistency problem is mitigated by raising p. Noticing this effect, each voter is more likely to vote for the incumbent party, raising p beyond a half even if the two parties are *ex ante* symmetrical.

We have focused on the case where the political parties have symmetric characteristics. An obvious extension of this research is to allow asymmetry in the parties' characteristics, such as preferences.

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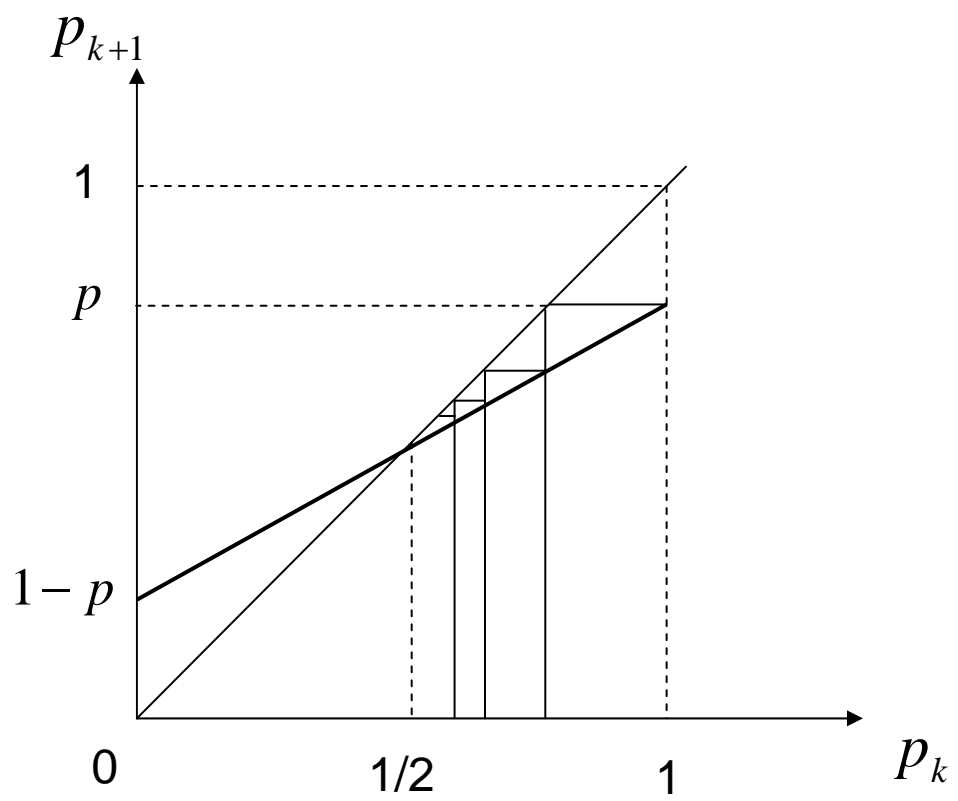
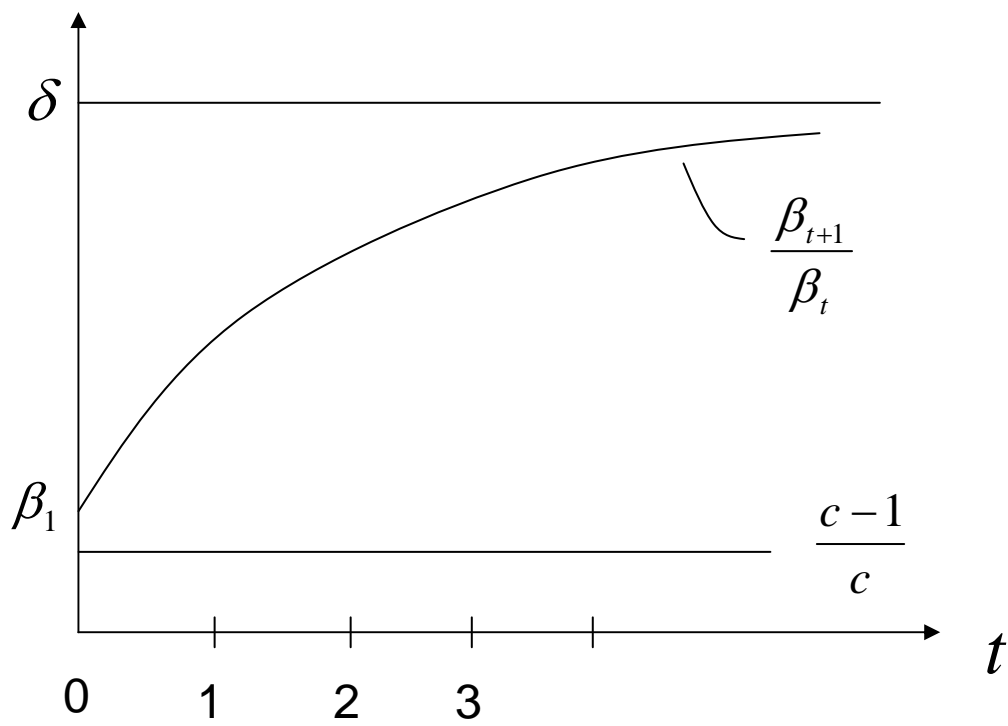Figure 1. The Transition of the Probability to be in Office
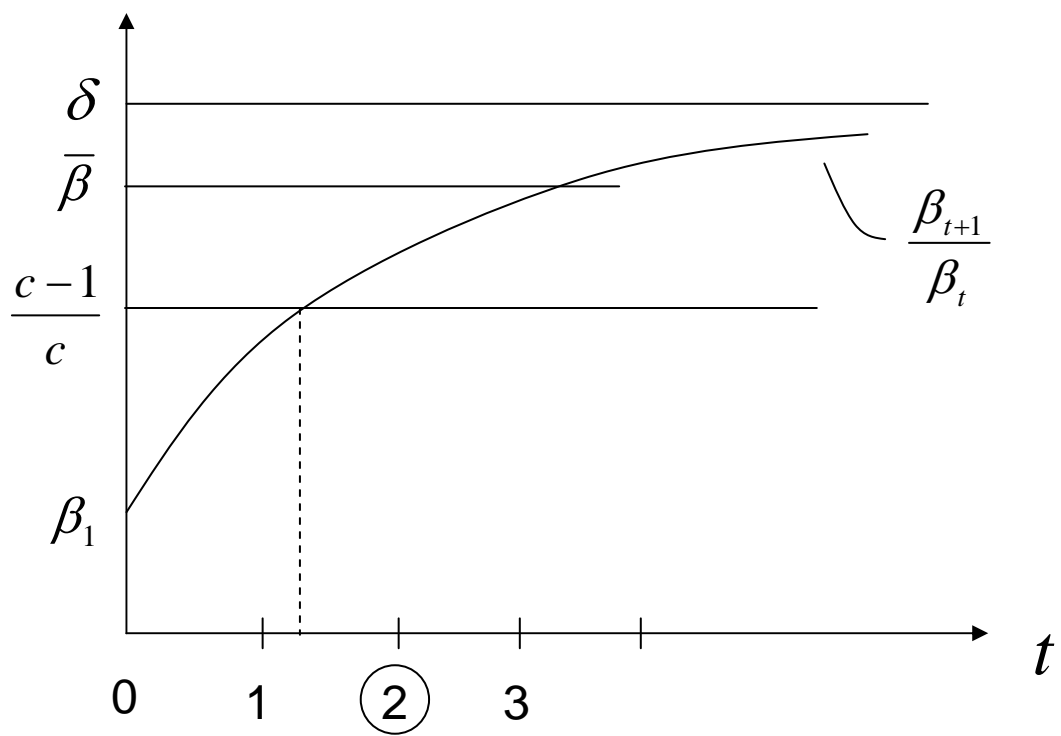
Figure 2. The Case of Small Implementation Costs

Figure 3. The Case of Medium Implementation Costs