積分 III
部分積分

戸瀬　信之

ITOSE PROJECT

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部分積分 (1)

開区間 (A, B) 上の関数

$$f: (A, B) \to \mathbf{R}, \quad g: (A, B) \to \mathbf{R}$$

連続.

と原始関数（不定積分）F, G を考えます：

$$F' = f, \quad G' = g$$

Leibnitz の公式から

$$(FG)' = F'G + FG' = fG + Fg$$

$$\left(\int fG\right)' = fG = (FG)' - Fg = (FG)' - \left(\int Fg\right)' = \left(FG - \int Fg\right)'$$

部分積分 (2)

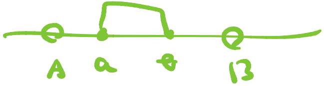
$$H_1'(t) - H_2'(t) = 0 \ (t \in (A,B)) \Rightarrow \exists C \in \mathbf{R} \, (H_1(t) - H_2(t) = C \ (t \in (A,B)))$$

なので

$$\left(\int fG\right)' = \left(FG - \int Fg\right)'$$

$$\sim - \sim = C$$

$$\int fGdt = FG - \int Fgdt$$



$$\int_a^b fGdt = [FG]_a^b - \int_a^b Fgdt$$

ここで F を f, G を g と表すと f が f', g が g' となるので

$$\boxed{\int_a^b f'gdt = [fg]_a^b - \int_a^b fg'dt}$$

部分積分.

計算例 (1)

(1)

$(e^t)' = e^t$

$$
\begin{aligned}
\int_0^1 te^t &= \int_0^1 t(e^t)'dt \\
&= \left[te^t\right]_0^1 - \int_0^1 d^t \cdot 1dt \\
&= e - \left[e^t\right]_0^1 \\
&= e - (e-1) = 1
\end{aligned}
$$

e^t

$(t)'$

$\int_0^1 e^t dt$

$=$

$[e^t]_0^1$

(2)

$$\int_1^2 \log t dt = \int_1^2 (t)' \cdot \log t dt$$
$$= [t \log t]_1^2 - \int_1^2 t \cdot \frac{1}{t} dt$$
$$= 2 \log 2 - \int_1^2 dt$$
$$= 2 \log 2 - 1$$

1.

$(\log t)'$

$\log 1 = 0.$

1.

(3)

$$
\begin{aligned}
\int_1^e t^2 \log t dt &= \int_1^e \left(\frac{t^3}{3}\right)' \log t dt \\
&= \left[\frac{t^3}{3} \log t\right]_1^e - \int_1^e \frac{t^3}{3} \cdot \frac{1}{t} dt \\
&= \frac{e^3}{3} - \frac{1}{3}\int_1^e t^2 dt \\
&= \frac{e^3}{3} - \frac{1}{3}\left[\frac{t^3}{3}\right]_1^e \\
&= \frac{e^3}{3} - \frac{1}{3} \cdot \frac{1}{3}(e^3 - 1) = \frac{2}{9}e^3 + \frac{1}{9}
\end{aligned}
$$

$(\log t)'$

$\log 1 = 0$

$\left(\frac{t^3}{3}\right)' = t^2.$

(4)

$(e^t)' = e^t$

$$\int_0^1 t^2 e^t dt = \int_0^1 t^2 (e^t)' dt$$

$$= \left[t^2 e^t\right]_0^1 - \int_0^1 2te^t dt$$

$$= e - 2\int_0^1 te^t dt = e - 2 \cdot 1 = e - 2$$