積分 V
置換積分（その 2）

戸瀬　信之

ITOSE PROJECT

V01 Nov 24, 2020 for CalcNT

$\rightrightarrows$ 変数変換.

$$x = \varphi(t)$$
$$dx = \varphi'(t)\,dt.$$

公式

代入. $x = \varphi(t)$

$$\int_A^B f(x)dx = \int_a^b f(\varphi(t))\varphi'(t)dt \quad (A = \varphi(a), B = \varphi(b))$$

$$\left[-\frac{1}{2}\cdot\frac{1}{x^2}\right]_1^2 = \int_1^2 \frac{dx}{x^3}$$

$$= \int_0^1 \frac{t}{(1+t^2)^3}\,dt = \frac{1}{2}\int \frac{(1+t^2)'}{(1+t^2)^3}\,dt$$

$\varphi(0) = 1$

$\varphi(1) = 2$

$(1+t^2)' = 2t.$

$x = \varphi(t) = 1 + t^2$

$f(x) = \dfrac{1}{x^3}$

(1)

$$I := \int_0^1 \frac{1}{1+x^2}\boxed{dx}$$

について考えます.

$$x = \varphi(t) := \tan t$$

とすると $\varphi'(t) = 1 + \tan^2 t$ で，対応

t	0	$\nearrow$	$\frac{\pi}{4}$
x	0	$\nearrow$	1

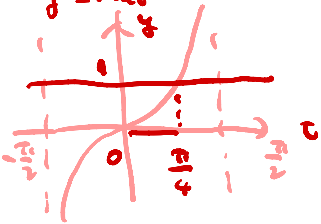
によって

$$I = \int_0^{\frac{\pi}{4}} \frac{1}{\cancel{1+\tan^2 t}} \cdot \boxed{\cancel{(1+\tan^2 t)}dt}$$

$$= \int_0^{\frac{\pi}{4}} dt = \frac{\pi}{4}$$

$(1+\tan^2 t)dt = dx.$

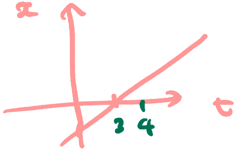
$y = \tan t$



$$(\tan t)' = \left(\frac{\sin t}{\cos t}\right)' = \frac{\cos t \cdot \cos t - \sin t(-\sin t)}{\cos^2 t} = \frac{\cos^2 t + \sin^2 t}{\cos^2 t} = \frac{1}{\cos^2 t}$$

$$= 1 + \frac{\sin^2 t}{\cos^2 t}$$

(2)



$$I := \int_{-2}^{1} \frac{x}{\sqrt{x+3}}\,dx = \int_{-2}^{1} \frac{x+3}{\sqrt{x+3}}\,dx - \int_{-2}^{1} \frac{3}{\sqrt{x+3}}\,dx$$

$$= \int_{-2}^{1} \sqrt{x+3}\,dx - 3\int_{-2}^{1} \frac{dx}{\sqrt{x+3}}$$

$x+3=t$ となるように $x=\varphi(t)=t-3$ とします. $\varphi'(t)=1$ で積分区間が

t	1	$\nearrow$	4
x	-2	$\nearrow$	1

と対応するので

$$I = \int_{1}^{4} \frac{t-3}{\sqrt{t}} \cdot 1dt$$

$$= \int_{1}^{4} \left(\sqrt{t} - \frac{3}{\sqrt{t}}\right) dt$$

$$= \left[\frac{2}{3}t\sqrt{t} - 3 \cdot 2\sqrt{t}\right]_{1}^{4} = \frac{2}{3}(8-1) - 6(2-1) = -\frac{4}{3}$$

$$(t\sqrt{t})' = \frac{3}{2}\sqrt{t} \to \left(\frac{2}{3}t\sqrt{t}\right)' = \sqrt{t}$$

$$(\sqrt{t})' = \frac{1}{2}\cdot\frac{1}{\sqrt{t}} \to (2\sqrt{t})' = \frac{1}{\sqrt{t}}$$

$$\left((x+3)^{\frac{3}{2}}\right)' = \frac{3}{2}(x+3)^{\frac{1}{2}}$$

$$\left(\frac{2}{3}(x+3)\sqrt{x+3}\right)' = \sqrt{x+3}.$$

$$\left((x+3)^{\frac{1}{2}}\right)' = \frac{1}{2}\cdot(x+3)^{-\frac{1}{2}}.$$

$$\left(2\sqrt{x+3}\right)' = \frac{1}{2}\frac{1}{\sqrt{x+3}}$$

具体例 (3)

$dx = 2t\,dt$

$\sqrt{x+3} = t$ となるように $x = \psi(t) = t^2 - 3$ とします．$\psi'(t) = 2t$ でで積分区間が

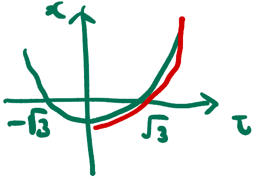
t	1	$\nearrow$	2
x	-2	$\nearrow$	1

と対応するので

$$= \int_{-2}^{1} \frac{x}{\sqrt{x+3}}\,dx. \qquad = t^2 - 3.$$

$t \geq 0$

$x + 3 = t^2$



$$I = \int_1^2 \frac{t^2-3}{\cancel{t}}\, 2\cancel{t}\,dt$$

$$= 2\int_1^2 (t^2-3)\,dt$$

$$= 2\left[\frac{t^3}{3} - 3t\right]_1^2 = \cdots\cdots$$

省略.

$$\int_0^{\frac{1}{2}} \sqrt{1-t^2}\,dt$$

$t = \sin\theta$