V1

(1) $\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \xi - \eta \\ \xi + \eta \end{pmatrix}$ から $\begin{cases} x + y = \sqrt{2}\,\xi \\ xy = \frac{1}{2}(\xi^2 - \eta^2) \end{cases}$

となる.

$$z = (x+y)^2 - xy - x - 2y$$

$$= 2\xi^2 - \frac{1}{2}(\xi^2 - \eta^2) - \frac{1}{\sqrt{2}}(\xi - \eta) - \frac{2}{\sqrt{2}}(\xi + \eta)$$

$$= \frac{3}{2}\xi^2 + \frac{1}{2}\eta^2 - \frac{3}{\sqrt{2}}\xi - \frac{1}{\sqrt{2}}\eta$$

となる

(2) $$z = \frac{3}{2}\left(\xi^2 - \frac{2}{\sqrt{2}}\xi\right) + \frac{1}{2}\left(\eta^2 - \frac{2}{\sqrt{2}}\eta\right)$$

$$= \frac{3}{2}\left(\xi - \frac{1}{\sqrt{2}}\right)^2 + \frac{1}{2}\left(\eta - \frac{1}{\sqrt{2}}\right)^2 - \frac{3}{4} - \frac{1}{4}$$

$$= \frac{3}{2}\left(\xi - \frac{1}{\sqrt{2}}\right)^2 + \frac{1}{2}\left(\eta - \frac{1}{\sqrt{2}}\right)^2 - 1 \geqq -1$$

(2) において等号成立は $\xi = \eta = \frac{1}{\sqrt{2}}$ である.

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

で最小値 $-1$ をとることが分かる.

VII $\vec{o} = A\begin{pmatrix}1\\0\end{pmatrix} = \vec{a}_1$, $\vec{o} = A\begin{pmatrix}0\\1\end{pmatrix} = \vec{a}_2$ から $A = (\vec{o}\ \vec{o}) = O_2$

となる.

VIII

(1) $$\begin{pmatrix} a_1 & c_1 \\ 0 & b_1 \end{pmatrix}\begin{pmatrix} a_2 & c_2 \\ 0 & b_2 \end{pmatrix} = \begin{pmatrix} a_1a_2 & a_1c_2 + c_1b_2 \\ 0 & b_1b_2 \end{pmatrix}$$

(2) $$\begin{pmatrix} a_1 & 0 \\ c_1 & b_1 \end{pmatrix}\begin{pmatrix} a_2 & 0 \\ c_2 & b_2 \end{pmatrix} = \begin{pmatrix} a_1a_2 & 0 \\ c_1a_2 + b_1c_2 & b_1b_2 \end{pmatrix}$$

(3) $$\begin{pmatrix} a_1 & 0 \\ 0 & b_1 \end{pmatrix}\begin{pmatrix} a_2 & 0 \\ 0 & b_2 \end{pmatrix} = \begin{pmatrix} a_1a_2 & 0 \\ 0 & b_1b_2 \end{pmatrix}$$

IX

(1) $$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \times \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} \\ -\begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} \\ \begin{vmatrix} 1 & 2 \\ 1 & -1 \end{vmatrix} \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$$

(2) $$\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \times \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} \\ -\begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix} \\ \begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix}$$

(3) $$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \times \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} \begin{vmatrix} 2 & 1 \\ 3 & -1 \end{vmatrix} \\ -\begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix} \\ \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} \end{pmatrix} = \begin{pmatrix} -5 \\ 7 \\ -3 \end{pmatrix}$$

X 直交射影を $\vec{w}$ とおきます.

(1) $\vec{w} = \dfrac{(\vec{a}, \vec{b})}{\|\vec{a}\|^2} \vec{a} = \dfrac{2}{3} \vec{a}$

(2) $\vec{w} = \dfrac{(\vec{a}, \vec{b})}{\|\vec{a}\|^2} \vec{a} = \dfrac{-1}{4} \vec{a}$

(3) $\vec{w} = \dfrac{(\vec{a}, \vec{b})}{\|\vec{a}\|^2} \vec{a} = \dfrac{2}{4} \vec{a} = \dfrac{1}{2} \vec{a}$

IX

(i) $\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} -1 & 2 \\ 3 & 1 \end{pmatrix} + \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$

$= \begin{pmatrix} -5 & 3 \\ 7 & 0 \end{pmatrix} + \begin{pmatrix} 5 & -4 \\ -4 & 5 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 3 & 5 \end{pmatrix}$

(ii) $\begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 2 & 2 \end{pmatrix}$

$= \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}\left(\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} + \begin{pmatrix} 3 & 1 \\ 2 & 2 \end{pmatrix}\right)$

$= \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 5 & 2 \\ 5 & 4 \end{pmatrix} = \begin{pmatrix} 15 & 10 \\ 15 & 10 \end{pmatrix}$

(iii) $\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 2 & 2 \end{pmatrix}$

$= \begin{pmatrix} 3 & 6 \\ 5 & 10 \end{pmatrix} + \begin{pmatrix} 7 & 5 \\ 7 & 5 \end{pmatrix} = \begin{pmatrix} 10 & 11 \\ 12 & 15 \end{pmatrix}$