積分 III

部分積分

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ITOSE PROJECT

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部分積分 (1)

開区間 (A, B) 上の関数

$$f : (A, B) \to \mathbf{R}, \quad g : (A, B) \to \mathbf{R}$$

と原始関数（不定積分）F, G を考えます：

$$F' = f, \quad G' = g$$

Leibnitz の公式から

$$(FG)' = F'G + FG' = fG + Fg$$

$$\left(\int fG\right)' = fG = (FG)' - Fg = (FG)' - \left(\int Fg\right)' = \left(FG - \int Fg\right)'$$

部分積分 (2)

$$H_1'(t) - H_2'(t) = 0 \ (t \in (A, B)) \Rightarrow \exists C \in \mathbf{R} \, (H_1(t) - H_2(t) = C \ (t \in (A, B)))$$

なので

$$\int fGdt = FG - \int Fgdt$$

$$\int_a^b fGdt = [FG]_a^b - \int_a^b Fgdt$$

ここで F を f, G を g と表すと f が f', g が g' となるので

$$\int_a^b f'gdt = [fg]_a^b - \int_a^b fg'dt$$

(1)

$$\begin{aligned}\int_0^1 te^t &= \int_0^1 t(e^t)'dt \\ &= \left[te^t\right]_0^1 - \int_0^1 e^t \cdot 1dt \\ &= e - \left[e^t\right]_0^1 \\ &= e - (e-1) = 1\end{aligned}$$

計算例 (2)

(2)

$$\begin{aligned}\int_1^2 \log t dt &= \int_1^2 (t)' \cdot \log t dt \\ &= [t \log t]_1^2 - \int_1^2 t \cdot \frac{1}{t} dt \\ &= 2\log 2 - \int_1^2 dt \\ &= 2\log 2 - 1\end{aligned}$$

計算例 (3)

(3)

$$
\begin{aligned}
\int_1^e t^2 \log t dt &= \int_1^e \left(\frac{t^3}{3}\right)' \log t dt \\
&= \left[\frac{t^3}{3} \log t\right]_1^e - \int_1^e \frac{t^3}{3} \cdot \frac{1}{t} dt \\
&= \frac{e^3}{3} - \frac{1}{3} \int_1^e t^2 dt \\
&= \frac{e^3}{3} - \frac{1}{3} \left[\frac{t^3}{3}\right]_1^e \\
&= \frac{e^3}{3} - \frac{1}{3} \cdot \frac{1}{3}(e^3 - 1) = \frac{2}{9} e^3 + \frac{1}{9}
\end{aligned}
$$

(4)

$$\begin{aligned}\int_0^1 t^2 e^t dt &= \int_0^1 t^2 (e^t)' dt \\ &= \left[t^2 e^t\right]_0^1 - \int_0^1 2te^t dt \\ &= e - 2\int_0^1 te^t dt = e - 2 \cdot 1 = e - 2\end{aligned}$$